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Design and Performance of LQR and LQR based Fuzzy Controller for Double Inverted Pendulum System

Narinder Singh Bhangal
Deptt. of Instrumentation & Control Engg.
NIT, Jalandhar, (PB) India
Email: singhn@nitj.ac.in

***Abstract*—The objective of this paper is to compare performance between two type of controller for a double inverted pendulum system. Double inverted pendulum is a non-linear, unstable and fast reaction system. DIP is stable when its two pendulums allocated in vertically position and have no oscillation and movement and also inserting force should be zero. The objective is to determine the control strategy that to delivers better performance with respect to pendulum angle's and cart position. In this paper LQR based fuzzy controller is designed and its performance is compared with Linear Quadratic Regulator controller using Matlab and Simulink. The results shows that LQR based fuzzy controller produced better response as compared to LQR control strategy.**

***Index Terms*—double inverted pendulum, LQR, fuzzy control, fusion function**

I. Introduction

The inverted pendulum offers a very good example for control engineers to verify a modern control theory. This can be explained by the facts that inverted pendulum is marginally stable, in control sense, has distinctive time variant mathematical model. The double inverted pendulum is a highly nonlinear and open-loop unstable system. The inverted pendulum system usually used to test the effect of the control policy, and it is also an ideal experimental instrument in the study of control theory [1], [2]. To stabilize a double inverted pendulum is not only a challenging problem but also a useful way to show the power of the control methods (PID controller, neural network, FLC, genetics algorithm, etc.). For fuzzy control of double inverted pendulum, a new idea of dealing with multivariate system is discussed [1]. Here composition error and composition rate of error is obtained by combining the fuzzy control theory and optimal control theory, but their responses are not discussed. Further research is focused on how to solve the problem of rule explosion [2]. Fuzzy control is then combined with artificial neural networks and to simplify the number of control rules [3]. A locally recurrent neural network was used to create a PID like neural network non linear adaptive controller for uncertain multivariable single input and multi output system [4]. A reconfigurable robust H∞ linear parameter varying controller is developed [5] A fusion function is designed using information fusion, six output variables of the system is synthesized as two variables error and change of error [6]-[8].But the performance and comparison of proposed controller with LQR is not made. This paper presents investigations of performance comparison between LQR and LQR based fuzzy control for a double inverted pendulum system. Performance of both controller strategies with respect to pendulums angles and cart position is examined.

The paper first provides a mathematical model for double inverted pendulum. Then LQR and LQR–Fuzzy controller based on state variables fusion to achieve the control of double inverted pendulum. The method of state variable fusion is used to build a new controller in order to reduce the number of input variable. The inverted pendulum system has many practical applications: to control the vertical deviation of a space shuttle during takeoff: to maintain a walking biped robot in its upright position (with a double spherical hip joint and knee joint): to balance a rocket as it moved to the launch pad etc.

II. Modeling of Double Inverted Pendulum

To control this system, its dynamic behavior must be analyzed first. The dynamic behavior is the changing rate of the status and position of the double inverted pendulum proportionate to the force applied. This relationship can be explained using a series of differential equations called the motion equations ruling over the pendulum response to the applied force. The double invnduerted pelum is shown in Fig. 1.

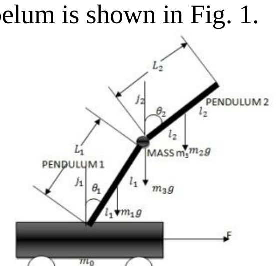

Figure 1. Schematic diagram of double inverted pendulum

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TABLE I. PARAMETERS OF INVERTED PENDULUM SYSTEM

$M(m_1,m_2,m_3)$	Mass of thecart,(first pole, second pole, joint) 5.8kg(1.5kg,.5kg,.75kg)
θ_1,θ_2	The angle between pole 1(2) and vertical direction (rad)
$L_1(l_1)$, $L_2(l_2)$	Length of pendulum first($2l_1$) and length of second pendulum ($2l_2$) ,1m,1.5m
g	Center of gravity 9.8m/s^2
F	Force applied to cart

The meanings and values of the parameters for inverted pendulum are given in Table I.

To derive its equations of motion, one of the possible ways is to use Lagrange equations [9]

$$\frac{d}{dt}\frac{dL}{d\dot{q}_i}-\frac{dL}{dq_i}=Q_i \tag{1}$$

where L = T - V is a Lagrangian, Q is a vector of generalized forces (or moments) acting in the direction of generalized coordinates q and not accounted for in formulation of kinetic energy T and potential energy V. Kinetic and potential energies of the system are given by the sum of energies of cart and pendulums.

$$\begin{aligned}T=&\frac{1}{2}(m_0+m_1+m_2+m_3)\dot{x}^2+(\frac{2}{3}m_1l_1+2m_2l_1+2m_3l_1)\dot{\theta}_1^2+\\&\frac{1}{6}m_2l_2\dot{\theta}_2^2+(m_1l_1+2m_2l_1+2m_3l_1)\dot{x}\dot{\theta}_1\cos\theta_1+2m_2l_2\dot{x}\dot{\theta}_2\cos\theta_2\\&+2m_2l_1l_2\cos(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2\end{aligned} \tag{2}$$

$$V=m_1gl_1\cos\theta_1+m_3gl_1\cos\theta_1+m_2g(2l_1\cos\theta_1+l_2\cos\theta_2) \tag{3}$$

Thus the Lagrangian of the system is given

$$\begin{aligned}L=&\frac{1}{2}(m_0+m_1+m_2+m_3)\dot{x}^2+(\frac{2}{3}m_1l_1+2m_2l_1+2m_3l_1)\dot{\theta}_1^2+\\&\frac{1}{6}m_2l_2\dot{\theta}_2^2+(m_1l_1+2m_2l_1+2m_3l_1)\dot{x}\dot{\theta}_1\cos\theta_1+2m_2l_2\dot{x}\dot{\theta}_2\cos\theta_2\\&+2m_2l_1l_2\cos(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2-m_1gl_1\cos\theta_1-m_3gl_1\cos\theta_1\\&-m_2g(2l_1\cos\theta_1+l_2\cos\theta_2)\end{aligned} \tag{4}$$

Differentiating the Lagrangian by and $\dot{\theta}$ and θ yields Lagrange equation (1) as:

$$\frac{d}{dt}\frac{dL}{d\dot{\theta}_1}-\frac{dL}{d\theta_1}=0 \tag{5}$$

$$\frac{d}{dt}\frac{dL}{d\dot{\theta}_2}-\frac{dL}{d\theta_2}=0 \tag{6}$$

Lagrange equation for the DICP system can be written in a more compact matrix form:

$$D(\theta)\ddot{\theta}+C(\theta,\dot{\theta})\dot{\theta}+G(\theta)=Hu \tag{7}$$

The stationary point of the system is $(x,\theta_1,\theta_2,\dot{x},\dot{\theta}_1,\dot{\theta}_2,\ddot{x})$ = (0, 0, 0, 0, 0, 0, 0), introduce small deviation around a stationary point and Taylor series expansion; the stable operated point of the double inverted pendulums are usually $\cos(\theta_1-\theta_2)=1$, $\sin(\theta_1-\theta_2)$, $\cos(\theta_1)\cong\cos(\theta_2)\cong 1$, $\sin(\theta_1)\cong\theta_1$, $\sin\theta_2\cong\theta_2$. Linearization is made at balance position; we can get the linear time invariant state space model [10].

$$\begin{bmatrix}\dot{x}\\\dot{\theta}_1\\\dot{\theta}_2\\\ddot{x}\\\ddot{\theta}_1\\\ddot{\theta}_2\end{bmatrix}=\begin{bmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&14.2545&-4.0090&0&0&0\\0&-14.2545&21.1077&0&0&0\end{bmatrix}\begin{bmatrix}x\\\theta_1\\\theta_2\\\dot{x}\\\dot{\theta}_1\\\dot{\theta}_2\end{bmatrix}+\begin{bmatrix}0\\0\\0\\1\\1.1818\\0.1818\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{bmatrix}\begin{bmatrix}x\\\theta_1\\\theta_2\end{bmatrix}+\begin{bmatrix}0\\0\\0\end{bmatrix}u(t) \tag{8}$$

III. DESIGN OF LINEAR QUADRATIC REGULATOR

We shall now consider the optimal regulator problem that, given the system equation.

$$\left.\begin{aligned}\dot{x}(t)&=Ax(t)+Bu(t)\\y(t)&=Cx(t)+Du(t)\end{aligned}\right\} \tag{9}$$

Determine the matrix K of the optimal control vector.

$$u(t)=-Kx(t) \tag{10}$$

So as to minimize the performance index

$$J=\int_0^t(x^T(t)Qx(t)+u^T(t)Ru(t))dt \tag{11}$$

where Q is a positive-semi definite and R is a positive-definite matrix. The matrices Q and R determine the relative importance of the error. Here the elements of the matrix K are determined so as to minimize the performance index.

Then $u(t)=-Kx(t)=-R^{-1}B^TPx(t)$ is optimal for any Initial x(0) state, where P (t) is the solution of Riccati equation, K is the linear optimal feedback matrix. Now we only need to solve the Riccati equation.

$$A^TP+PA-PBR^{-1}B^TP+Q=0 \tag{12}$$

where Q and R chose as Q = diag ([10 60 80 0 0 0]) and R = 1.

Therefore,

K= $-R^{-1}B^TP$ = [10 275.2453 -515.6502 16.2044 22.1046 -111.9285]

IV. LQR-FUZZY LOGIC CONTROLLER DESIGN

The double inverted pendulum has six state variables: x, $\dot{x}$, θ_1, $\dot{\theta}_1$, θ_2, $\dot{\theta}_2$, if we use the normal fuzzy control method, the number of rules is equal to 7^6. This can cause "rule explosion". To solve this problem, optimal control theory and fuzzy control strategy are combined. We transform multiple variables into a comprehensive error E and the rate of change of error EC, which greatly simplified the FLC controller. The structure of the controller is shown Fig. 2.

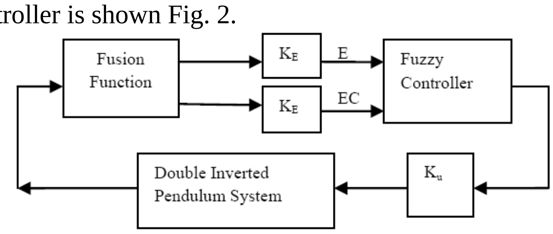

Figure 2. The structure of fuzzy logic controller

For the double inverted pendulum:

$$F_1(X) = \begin{bmatrix} \frac{K_1}{K_3} & \frac{K_2}{K_3} & \frac{K_3}{K_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{K_4}{K_6} & \frac{K_5}{K_6} & \frac{K_6}{K_6} \end{bmatrix} \qquad (13)$$

Hence, comprehensive error E and the rate of change of error EC can be calculated:

$$\begin{bmatrix} E \\ EC \end{bmatrix} = F_1(X)X^T$$

In this case

$$E = K_x.x + K_{\theta_1}.\theta_1 + K_{\theta_2}.\theta_2 ,$$

$$EC = K_{\dot{x}}.\dot{x} + K_{\dot{\theta}_1}.\dot{\theta}_1 + K_{\dot{\theta}_2}.\dot{\theta}_2 \qquad (14)$$

The coefficient of colligation, $K = \left[K_x \ K_{\theta_1} \ K_{\theta_2} \ K_{\dot{x}} \ K_{\dot{\theta}_1} \ K_{\dot{\theta}_2}\right]^T$, is chosen as the state feedback coefficient in LQR design.

According to the parameter of the double inverted pendulum, the fusion function can be calculated.

$$F_1(X) = \begin{pmatrix} -0.0854 & -0.5479 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5144 & -0.0744 & 1 \end{pmatrix}$$

The membership functions are triangular and input and output variables have seven membership functions each as shown in the Fig. 4, Fig. 5 and Fig. 6.Schematic diagram of the inverted pendulum system with LQR-Fuzzy logic controller is as shown in the Fig. 3.

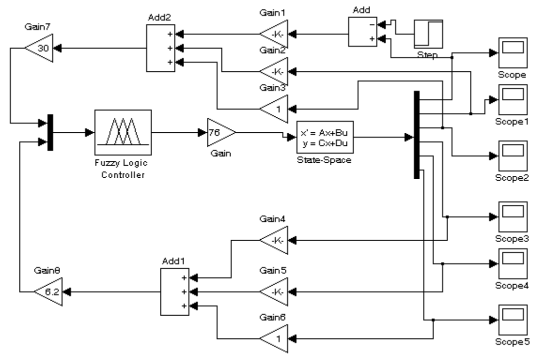

Figure 3. Block diagram of inverted pendulum System with LQR-FLC controller

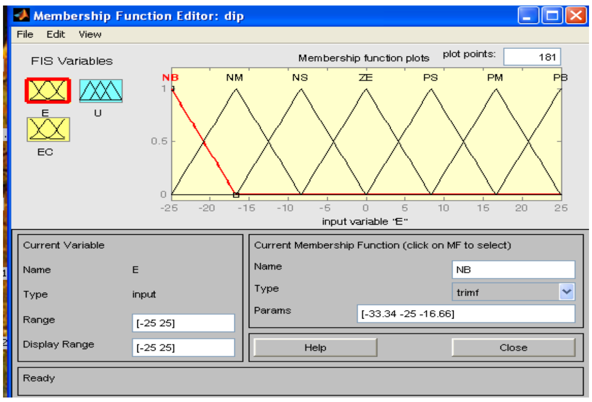

Figure 4. The membership functions of variable E

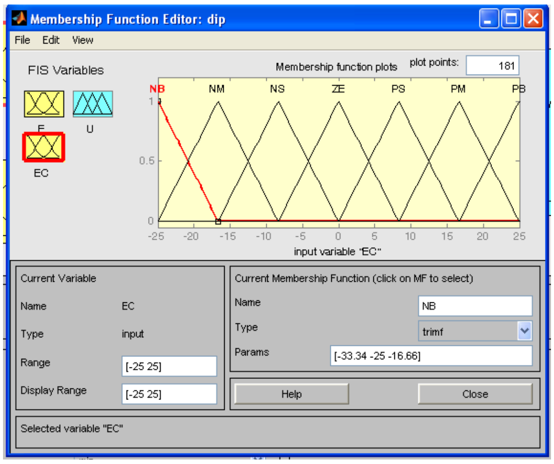

Figure 5. The membership functions of variable EC

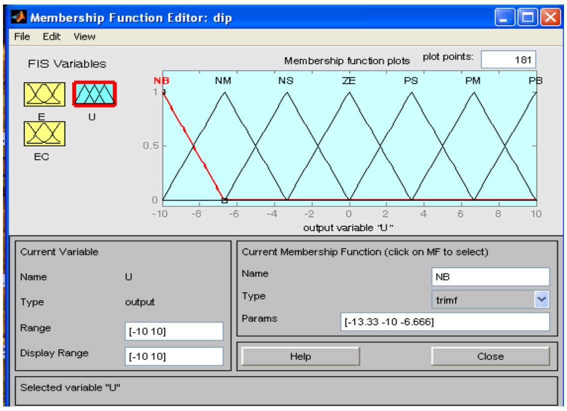

Figure 6. The membership functions of output variable U

V. SIMULATION AND RESULT

The system under consideration and the proposed controllers are modeled and simulated in the MATLAB/Simulink environment. The step response performance of the two controllers is compared. Fig.7 shows the step response of the system with LQR controller. The response of cart, lower and upper pendulum with fuzzy controller is as shown in the Fig. 8, Fig. 9 and Fig. 10.

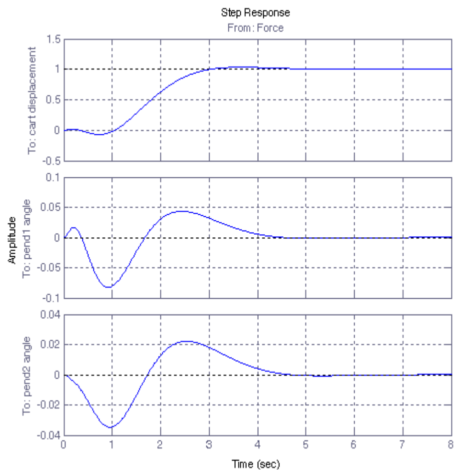

Figure 7. The step response of Double Inverted Pendulum

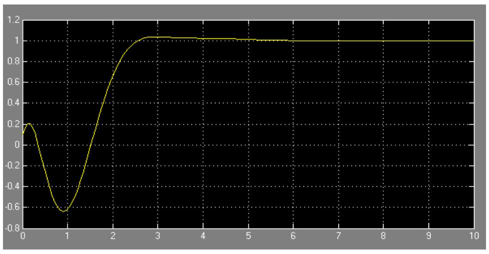

Figure 8. The step response of cart position

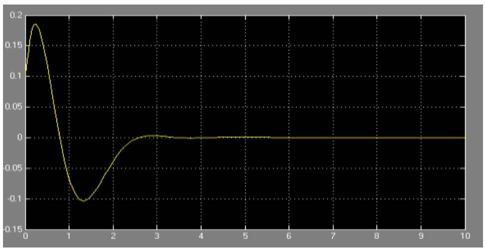

Figure 9. The step response of lower pendulum

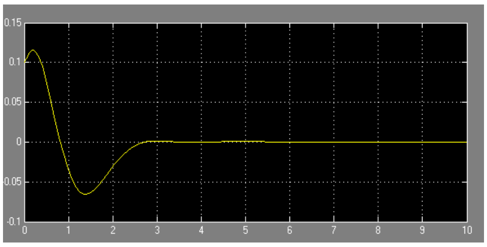

Figure 10. The step response of upper pendulum

TABLE II. THE PERFORMANCE CHARACTERISTICS OF CART POSITION

Time response specification	LQR controller	LQR-FUZZY controller
Settling time (T_s)	3sec	2.5sec
Peak overshoot	0%	0%
steady state error (e_{ss})	0	0

TABLE III. THE PERFORMANCE CHARACTERISTICS OF LOWER PENDULUM

Time response specification	LQR controller	LQR-FUZZY controller
Settling time (T_s)	4.3SEC	3SEC
Peak overshoot	8%	10%
Steady state error (e_{ss})	0	0

TABLE IV. THE PERFORMANCE CHARACTERISTICS OF UPPER PENDULUM

Time response specification	LQR controller	LQR-FUZZY controller
Settling time (T_s)	4.5SEC	2.5SEC
Peak overshoot	3%	6%
Steady state error (e_{ss})	0	0

The performances of the system under consideration equipped with the proposed controllers are given in Table II, Table III and Table IV.

From both controller LQR and fuzzy controller's result, It is clear that both are successfully designed but LQR-Fuzzy controller exhibits better response and performance.

VI. CONCLUSION

In this paper, LQR and LQR-Fuzzy controller are successfully designed for double inverted pendulum system. Based on the results, both controllers are capable of controlling the double inverted pendulum's angles and the cart position of the linearized system. However, the simulation result shows that LQR based fuzzy controller has a better performance as compared to the LQR controller in controlling the double inverted pendulum system.

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Narinder Singh Bhangal has done his B. Tech in Electrical Engg. from Punjab University, Chandigarh, India in1984 and did his M. Tech in control systems from Punjab Agricultural University, Ludhiana ,Punjab, India .Currently working as Associate Professor at National Institute of Technology, Jalandhar, Punjab. His area of research is optimal control, fuzzy, neuro-fuzzy control and currently doing Ph.D in robust control of flexible manipulators.