Design of Multilayer Optical Filters Using the Fourier Transform Approach

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***Abstract*—This paper presently the design of multilayer optical filters using the Fourier Transform approach. In this study, an analytical relationship has been established between the spectra and the refractive index while keeping the optical thickness of the filter layers constant. The outcome of this design is a continuous refractive index curve, which was then transformed into a discrete curve, thus enabling the direct design of multilayer optical filters.**

***Index Terms*—multilayer optical filters, fourier transform, fourier spectrum, thin films**

I. Introduction

Nowadays, with the advancement of optical processing as well as wide application of optical devices in various scientific and industrial fields, there are strong requirements for development of robust Computer-Aided design (CAD) tools to efficiently design such devices.

Thin film optical filters are one of the most important components in this area. These filters could be applied in eye glasses, window glass, lamps, cold mirrors, detectors and optical components used for telecommunication and optical processing. Optical filters are generally divided into five categories [1]: anti-reflective coatings, reflective coatings, edge filters, optical band pass filters, and optical beam deflectors.

A multilayer filter is consists of several thin layers (compatible with wavelength ranges) of various dielectric materials that have been deposited on a substrate. Thus, a filter is specified by determining three parameters namely, the number of layers as well as the refractive index and the thickness of each layer. The aforementioned parameters should be determined in a way that the reflected spectrum or the spectrum passed from the filters should meet the desired requirements as close as possible.

There are several approaches to design such filters like Differential Correction [1], [2], Graphical [2], Electrical [1], [2], Analytical [1]-[3], Special [2], and Merit Function methods [1]. Each of the aforementioned techniques has its own advantages and disadvantages, making them suitable for specific kind of filters.

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The Fourier Transform method is an analytical direct design method that is most suitable for the design of anti-reflective coatings. Because of its simplicity and speed [3], this method has been considered widely by researchers [4], [5].

II. Fourier Transform Method

Multilayer filter design approaches using Fourier methods can be classified into two categories. In the first category, referred to as "indirect methods", the first step is to design a heterogeneous filter. Then, a discretization process is performed on the response of the refractive index of the heterogeneous filter [4], [5]. However, due to the approximations to be made in both design phases, these methods are not suitable, especially when the number of layers is low.

In the second category, referred to as "direct methods", a multilayer filter is directly designed. Such methods are efficient for filters with low number of layers, since they require solving several systems of equations.

However, the existing direct methods [2], [6], cannot practically be used due to high degree of approximation and lack of robust algorithm for implementation on computer.

A. Direct Design Method

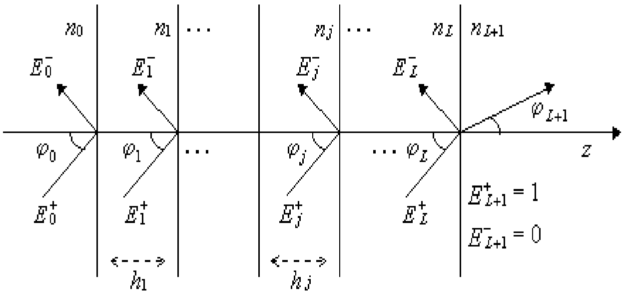

Figure 1. L-Layer filters and electrical field components.

Primary research regarding the direct design method has been conducted by Pegis and Delano [2]. Starting from Maxwell's laws and writing relations for the *L*-layer system of Fig. 1, the relationship between the electric field components can be expressed as

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$$\begin{bmatrix} E_{j-1}^{+} \\ E_{j-1}^{-} \end{bmatrix} = \frac{1}{t_{j-1}} \begin{bmatrix} e^{-ig_j} & r_{j-1} e^{ig_j} \\ r_{j-1} e^{-ig_j} & e^{ig_j} \end{bmatrix} \times \begin{bmatrix} E_j^{+} \\ E_j^{-} \end{bmatrix} \quad (1)$$

where E_j^{+} and E_j^{-} are the reciprocating components of the electrical field at interface j.

r_j and t_j are respectively the reflection and transmission coefficient corresponding to the domain and given by

$$r_{j-1} = \frac{u_{j-1} - u_j}{u_{j-1} + u_j}, \ t_{j-1} = 1 + r_{j-1}, \ j = 1, 2, \ldots, L \quad (2)$$

And

$$u_j = \begin{cases} n_j \cos\varphi_j & TE \text{ modes} \\ n_j \sec\varphi_j & TM \text{ modes} \end{cases} \quad (3)$$

where n_j is the refractive index of layer j. The efficient optical phase g_j is given by

$$g_j = \frac{2\pi}{\lambda} n_j h_j \cos\varphi_j \quad (4)$$

where h_j and φ_j represent the thickness and the radiation angle of layer j, respectively. λ is the wavelength.

Thus, the reflection and transmission coefficients, respectively T and R, can be expressed as

$$T = \frac{n_{L+1} \cos\varphi_0}{n_0 \cos\varphi_{L+1}} \left| \frac{1}{E_0^{+}} \right|^2 \ R = \left| \frac{E_0^{-}}{E_0^{+}} \right|^2 \quad (5)$$

where n_o is the refractive index of the medium and n_{L+1} the refractive index of the substrate.

In the Fourier Transform Method, another parameter is also defined as follows:

$$\rho = \sqrt{\frac{u_0}{u_{L+1}}} e^{-iG} E_0^{-}, \ G = \sum_{j=1}^{L} g_j \quad (6)$$

When the radiation is assumed to be vertical, (φ_j=0) this parameter is related to T and R by

$$\frac{R}{T} = |\rho|^2 \quad (7)$$

It is evident that the type of polarization does not affect the vertical radiation. Therefore, the value of ρ is obtained by solving (1)

$$\rho = \frac{1}{f} \sum_{k=0}^{L} a_k e^{-2ikg} \quad (8)$$

where,

$$f = \prod_{k=0}^{L} (1 - r_k)^2 \quad (9)$$

And

$$a_k = r_k + F_k(r_0, r_1, \ldots, r_L) \quad (10)$$

$F_k(0)$ is a nonlinear function which will be described in the next section. It is to noted that when we are obtaining ρis that the optical phases are assumed to be equal (g_j=g).

This is the main restriction in using the Fourier Transform Method and as a result, the refractive index multiplied by the geometric thickness of each layer (which is called the optical thickness) remains constant. Then, we need to calculate $|\rho|^2$ as

$$|\rho|^2 = \frac{1}{f^2} \left[b_0 + 2\sum_{k=1}^{L} b_k \cos(2kg) \right] \quad (11)$$

With

$$b_k = \sum_{s=0}^{L-k} a_s \, a_{s+k} \quad (12)$$

To estimate f^2, let us reformulate Equation (11) as [1]

$$|\rho|^2 = c_0 + 2\sum_{k=1}^{L} c_k \cos(2kg), \ c_k = \frac{b_k}{f^2} \quad (13)$$

Therefore, we can determine the Fourier coefficients of the Fourier spectrum (R/T) or the values of c_k.

Accordingly, the coefficients a_k can be deduced by solving the system Equation (12).

Then, the reflection coefficients of the scope will be determined by solving the system Equation (10) and then, considering that n_0 and n_{L+1} are known and by employing (2), the rest of the refraction coefficients can be obtained.

The thickness of layers can be obtained from $n_j h_j = \lambda_0/4$ assuming that the optical thicknesses of thin films are constant.

B. *Indirect Design Method*

Among those who have conducted significant studies on direct design method, we can refer to Verly [4], [7], [8], Dobrowolski [3], [5], [9], [10], and Sossi [10]-[15].

The basic equation used to design a homogeneous filter is as follows:

$$\ln \frac{n(x)}{\sqrt{n_0 n_{L+1}}} = \frac{i}{\pi} \int_{\infty}^{-\infty} \frac{Q(\sigma)}{\sigma} \exp(-2i\pi\sigma x) d\sigma \quad (14)$$

where $n(x)$ is the refractive index according to the optical gap$x$; σ=1/λ is proportional to frequency and $Q(\sigma)$ is a complex function known as the Fourier Spectrum, defined differently as

$$|Q_1| = \sqrt{\frac{1}{2}\left(\frac{1}{T} - T\right)} \quad (15)$$

$$|Q_2| = \sqrt{\frac{R}{T}} \quad (16)$$

Or

$$|Q_3| = \sqrt{1 - T} \quad (17)$$

Depending on the specific type of filter which is designed, we can notice that the relationship between the Fourier spectrum and the filter parameters is similar to the Fourier Transform relationship. Therefore, this method is known as the Fourier method.

III. Proposed Improving Algorithm for Direct Method

To improve the direct approach of the Fourier Transform method, we introduced an efficient technique

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to solve the nonlinear system constituted by Equations (10) and (12) and we used a gradient algorithm to adjust the scope of the reflection and refraction coefficients as well as the layer thicknesses. After obtained the C_k coefficients and calculating the Fourier Spectrum $|\rho|^2$, we estimated the value of f^2 [2]:

$$f^2 = \frac{1}{\gamma_0}[1 + \frac{1}{\gamma_0^6}\sum_{k=1}^{L}(\gamma_k \sum_{j=0}^{L}\gamma_j^2 - \gamma_0 \sum_{s=1}^{L-k}\gamma_s\gamma_{s+k})^2] \quad (18)$$

With

$$\gamma_0 = 1 + c_0 \text{ and } c_k = \gamma_k,\ k = 1, 2, \ldots, L \quad (19)$$

To numerically solve this system, let us consider the case of a 4-layer filter (L = 4). The coefficients a_k of the Fourier Transform are related to the b_k coefficients as

$$\begin{aligned} a_0^2 + a_1^2 + a_2^2 + a_3^2 + a_4^2 &= b_0 \\ a_0\, a_1 + a_1\, a_2 + a_2\, a_3 + a_3\, a_4 &= b_1 \\ a_0\, a_2 + a_1\, a_3 + a_2\, a_4 &= b_2 \\ a_0\, a_3 + a_1\, a_4 &= b_3 \\ a_0\, a_4 &= b_4 \end{aligned} \quad (20)$$

With

$$b_k = c_k f^2 \quad (21)$$

Solving this system results in 2^L sets of combinations (2^4 in the present example) while only one is the correct solution. To solve the aforementioned system, let us reformulate it as

$$A * \vec{a} = \vec{b} \quad (22)$$

With

$$\vec{a} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_L \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_L \end{bmatrix} \quad (23)$$

And

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_L \\ 0 & a_0 & a_1 & \cdots & a_{L-1} \\ 0 & 0 & a_0 & \cdots & a_{L-2} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & a_0 \end{bmatrix} \quad (24)$$

To solve this matrix equation, a primary random vector $\vec{a}$ is first chosen. Accordingly,

$$\vec{a} = A^{-1} * \vec{b} \quad (25)$$

Thus obtaining a new value for vector $\vec{a}$. An iterative process is then performed. If the initial solution is viable, the procedure will be converging to a proper solution. If the initial value is not viable, i.e., if the error increases, a new initial value will be selected automatically.

After getting the coefficients a_k, the system (10) is solved. In the present example, we have

$$\begin{aligned} a_0 &= r_0 \\ a_1 &= r_1 + \overbrace{r_0 r_1 r_2 + r_0 r_2 r_3 + r_0 r_3 r_4}^{F_1(r_0 \ldots r_L)} \\ a_2 &= r_2 + r_0 r_1 r_3 + r_0 r_2 r_4 + r_1 r_2 r_3 + r_1 r_3 r_4 + r_0 r_1 r_2 r_3 r_4 \\ a_3 &= r_3 + r_2 r_3 r_4 + r_1 r_2 r_4 + r_0 r_1 r_4 \\ a_4 &= r_4 \end{aligned} \quad (26)$$

This set of equations, especially when the number of layers is high, becomes very complex to solve. Thus, it has been assumed that $|r_k|<1$ [2] and accordingly neglected the terms with high degrees, leading to $a_k=r_k$.

The important issue while implementing this system is how we determine each equation. The procedure of determining the different terms of the functions $F_k(0)$ is highly time-consuming.

Here, we explain this procedure by giving an example, if we assume that the number of layers is 5 (L=5) and we aim to determine the equation related to a_3 (k=3). Firstly, we specify all binary numbers which have the length of 5 and contain 3 numbers of 1. These numbers include: {00111, 01011, 01101, 01110, 10011, 10101, 10110, 11001, 11010, 11100}

The number of these terms in general equals:

$$\binom{L}{k} = \frac{L!}{(L-k)!\,k!} \quad (27)$$

It is to be noted that in the aforementioned example, it equals 10. Then we add 1 to the beginning of each term and a 0 at the end of each. For example 00111 become 1001110. Then, we examine each term from the beginning to the end and in case it changes at any stage, we write the r_j index (j) equal to the number referring to that stage.

For example, for 1001110, it becomes $r_0 r_2 r_5$ that has been obtained by employing the pattern, $1 \xrightarrow{r_0} 0 \rightarrow 0 \xrightarrow{r_2} 1 \rightarrow 1 \rightarrow 1 \xrightarrow{r_5} 0$.

The number of stages in this example equals 6 (that is: L+1=6). It begins from stage 0 and ends at stage 5. If we employ this method, the $F_3(.)$ function becomes:

If we use this method, the function becomes:

$$\begin{aligned} F_3(r_0, r_1, r_2, r_3, r_4, r_5) &= r_0 r_2 r_5 + r_0 r_1 r_2 r_3 r_4 r_5 + \\ r_0 r_1 r_3 r_4 r_5 + r_0 r_1 r_4 &+ r_1 r_3 r_5 + r_1 r_2 r_3 r_4 r_5 + r_1 r_2 r_4 + r_2 r_4 r_5 + r_2 r_3 r_4 \end{aligned} \quad (28)$$

After obtaining the value of reflection coefficients r_k, a gradient algorithm is used in order to adjust these values as well as the refraction coefficients and thicknesses. This simple procedure is very effective in reducing errors and deficiencies existed in the earlier stages.

The improvement procedure may be repeated to obtain the desired level of error. Finally, the fabricated filter is compared to the desired filter and the root mean squared error between the two is specified. The flowchart of this program is shown in Fig. 2.

The aforementioned program has been used in the design of several filters and high speed and avoiding

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approximations used in direct methods are among the advantages of this program.

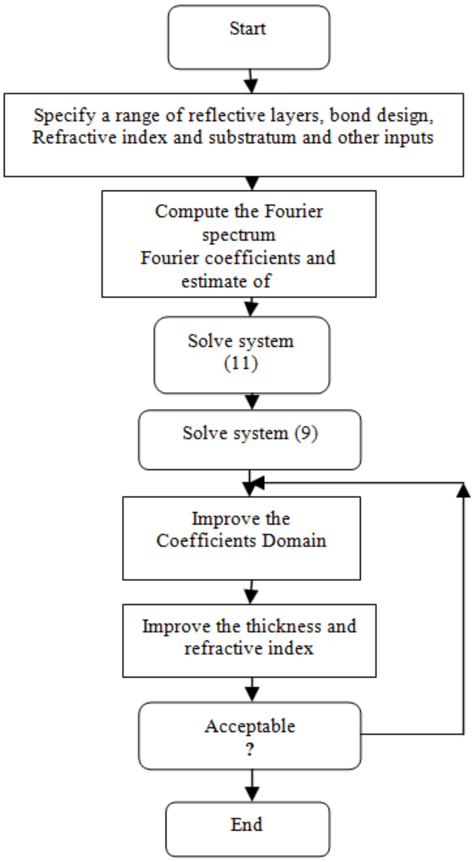

Figure 2. Flowchart of the proposed algorithm.

IV. Results

In this part, two examples have been presented:

1) Test 1; here we aim to design a band-stop filter R=0 in wavelengths bands from λ_{Low}=1μ*m* to λ_{High}=2.1μ*m*.

Light enters the filter from an environment with a refractive index of n_0=2.35 and exists to an environment with a refractive index of n_{L+1}=1.35. This filter is used as an anti-reflective coating on laser equipments.

Solution: After running the program on a 166MHz Pentium computer, the following results were obtained:

The persecution time of the program was 502 seconds and the Root Mean Squared Error was calculated to be RMSE=0.0023. The results are presented in Fig. 3, and Table I.

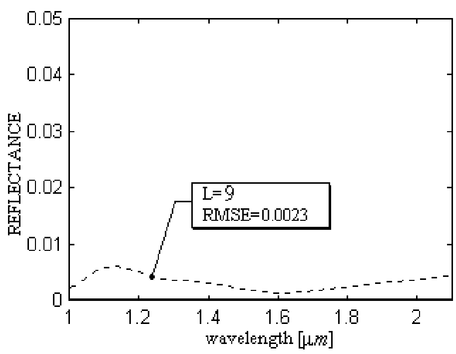

Figure 3. Reflexive spectrum of anti-reflective coatings

In addition, we can easily transform the refractive index into a multi layer curve using Herpin Equivalent Layer Method [12].

TABLE I. Results of Example 1

Layer	**Refractive index**	*Wavelength μm*
1	**2.337**	*0.1573*
2	**2.346**	*0.1735*
3	**2.343**	*0.1029*
4	**2.290**	*0.1331*
5	**2.158**	*0.1589*
6	**1.975**	*0.2247*
7	**1.847**	*0.1962*
8	**1.662**	*0.2075*
9	**1.376**	*0.2414*

2) Test 2; here we aim to design a band-stop filter R=0.95 in wavelengths bands from λ_{Low}=2.3μ*m* to λ_{High}=2.7μ*m*, with (BW = 0.4). Light enters the filter from an environment with a refractive index of n_0=2.35 and exists to an environment with a refractive index of n_{L+1}=1.0. Acceptable error is assumed to be 10%.

Solution: After running the program, the following results were obtained:

The persecution time of the program was 2061 seconds and the Root Mean Squared Error was calculated to be RMSE=0.10. The final spectrum and results are presented in Fig. 4 and Table II.

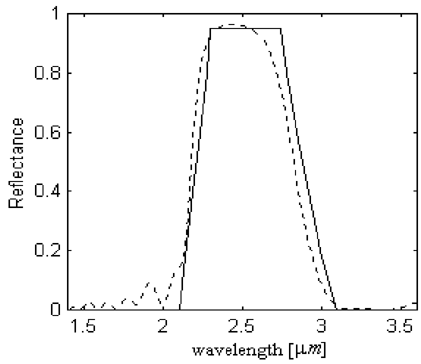

Figure 4. Desired (solid line) and calculated (dashed line) reflectance spectrum.

TABLE II. Results of Example 2

Layer	**Refractive index**	*Wavelength μm*
1	**1.788**	*0.3500*
2	**2.507**	*0.2609*
3	**2.044**	*0.2336*
4	**2.678**	*0.2334*
5	**1.953**	*0.3200*
6	**2.826**	*0.2211*
7	**2.004**	*0.3118*
8	**2.870**	*0.2146*
9	**1.980**	*0.3157*
10	**2.847**	*0.2130*
11	**2.073**	*0.3027*
12	**2.724**	*0.2206*
13	**2.091**	*0.2989*
14	**2.470**	*0.2450*
15	**1.980**	*0.3157*
16	**2.245**	*0.2716*
17	**1.894**	*0.2514*
18	**1.843**	*0.3504*
19	**1.572**	*0.3976*

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V. Conclusion

In this paper, after studying the Fourier Transform Method in the design of multilayer optical filters, we classified this method into two categories of direct and indirect approaches, based on previous research results.

Unlike the indirect approach, that several algorithms have been provided for its implementation on commuters, no algorithm has been yet presented for the implementation of the direct approach. After offering an algorithm for this approach and employing it for the design of filters with various spectrums (that two of them have already been reviewed), it was inferred that this method of design has many advantages compared to other methods, some of those advantage include the high speed of design and accuracy. In further studies, we aim to find a suitable approach to improve the results of design.

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