Generalized Erosion and Dilation Operations of Distance Functions for Deforming Soft Objects

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***Abstract*—In soft object modeling, a complex soft object is obtained by performing blending operations on primitive soft objects and primitive soft objects determine the shape of the complex soft object. To make the shapes of primitive soft objects more diverse, this paper proposes generalized erosion and dilation operations of distance functions to deform a primitive soft object. The proposed operations are able to increase and decrease the influential radius of the distance function of a soft object repeatedly via a sequence of dilating and eroding distance functions. Thus, unlike affine transform which deforms a surface by matrix multiplication, the proposed operations deform (dilate or erode) a soft object by a chosen dilating or eroding function, called dilation or erosion. In addition, because the proposed operations also allow more than two dilating and eroding functions to deform a soft object sequentially and successively, partial erosion and dilation, double or triple partial erosions and dilations are also allowed for local deformation. Thus, by using the proposed operations as a new distance function to redefine a soft object, a primitive soft object can be deformed for obtaining the following special deformation effects: concave, convex, point-dilation, disk-dilation, point-erosion, disk-erosion, four-ball-shaped, six-ball-shaped and UFO-shaped effects.**

***Index Terms*—implicit surface, field function, soft object modeling, distance function**

I. Introduction

In implicit surface modeling, a primitive implicit surface is defined as a level surface of defining functions. Furthermore, primitive implicit surfaces can be connected smoothly by blending operations, such as union, intersection and difference operations in [1]-[8], for creating a more complex surface.

Among modeling techniques of implicit surfaces, soft object modeling especially uses field functions as defining functions to define primitive soft objects. Because a field function is required to decrease from 1 to 0 within an influential region, primitive soft objects can be blended easily by performing addition operation only. Precisely, a field function is defined as a composition of a potential function and a distance function. Potential function ensures a field function to be decreasing to zero [9], [10]. Distance function determines and controls the size and the shape of a primitive soft object, so distance functions with different shapes were developed, such as spheres [10], super-ellipsoids and super-quadrics [11], generalized distance functions [12], and sweep objects.

In addition, to make the shape of a primitive soft object more diverse, deforming a soft object is a good method, too. Regarding this, affine transform was applied on implicit surfaces in [13] and applied to soft object in [14] for deformation like translation, rotation, shearing, scaling, tendering and twisting. In this method, a transformation matrix is defined to specify the deformation factors first, and then position vector has to be multiplied by the transformation matrix before be used to calculate the defining function.

On the other hand, unlike affine transform using matrix multiplication, erosion operation and dilation operation of distance functions were also proposed to deform a soft object [15] by eroding or dilating a soft object through an operation of the distance function defining the soft object and a chosen eroding or dilating function, respectively. But unfortunately, the two operations in [15] deform (enlarge or shrink) a soft object totally and globally, and only binary erosion or binary dilation operations were developed To conquer these problems and offer partial erosion and partial dilation with local deformation for a soft object, this paper proposes generalized erosion and dilation operations of distance functions. The newly proposed generalized operations integrate more than two erosion or dilation operations together into a single operation, as a result:

- A primitive soft object can be eroded (shrunk) and dilated (enlarged) sequentially, repeatedly and interchangeably through a sequence of eroding and dilating distance functions. That is, double, triple, and multiple erosions and dilations can be obtained.
- A primitive soft object can be eroded and dilated locally, not totally. That is, partial erosion and dilation, and even double or triple partial erosions and dilations can be generated.
- In addition, more deformation effects can be created.

The remainder of this paper is organized as follows. Section II reviews related works about soft objects. Section III describes the problems of erosion and dilation operations of distance functions. Section IV presents generalized erosion and dilation operations of distance functions. Section V demonstrates some deformation effects via the proposed operations. Conclusion is given in Section VI.

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II. Related Works of Soft Object Modeling

Sections *A-C* review soft object modeling. Affine transform is described in Section *D*.

A. Soft Objects

Let $f_i(X):R^3 \rightarrow R_+$ be a field function, $i=1, 2, \ldots$ Then, a primitive soft object is defined as the point set:

$$\{X \in R^3 | f_i(X)=0.5\}$$

where $X \equiv (x, y, z) \in R^3$ and $R_+ \equiv [0, \infty]$. $f_i(X)>0.5$ represents the inside of the object and $f_i(X)<0.5$ the outside $f_i(X)=0.5$ the surface (shape). In the following, the point set is denoted as $f_i(X)=0.5$ for short.

In fact, a field function $f_i(X)$ is usually written as a composition of a potential function $P(d)$ and a distance function $d_i(X)$ by:

$$f_i(X)=(P \circ d_i)(X)= P(d_i(X))$$

In addition, $P(d):[0,1] \rightarrow [1,0]$ decreases from 1 to 0 as d increases from 0 to 1 and $P(0.5)=0.5$. Many potential functions were proposed with special purposes in [9], [10]. In fact, $d_i(X)$ can be defined using an closed surface $d_i(X)=1$, called influential region, and is written by:

$$d_i(X) = r / R_d = \|\overrightarrow{oX}\| / \|\overrightarrow{oI}\|$$

where $r=\|\overrightarrow{oX}\|=(x^2+y^2+z^2)^{0.5}$ and $R_d =\|\overrightarrow{oI}\|$ is called the influential radius of influential region $d_i(X)=1$ for X, i.e. the distance from the origin to the intersecting point of the vector $X=(x, y, z)$ with the influential region $d_i(X)=1$, as shown in Fig. 1. R_d also indicates that the value of $P(d_i(X))$ will drop to zero when r reaches R_d. Here subscript d is used to indicate that R_d is the influential radius calculated within influential region $d(X)=1$. Due to $P(0.5)=0.5$, the shape of $(P \circ d_i)(X)=0.5$ is similar to the shape $d_i(X)=0.5$, and hence influential region $d_i(X)=1$ determines the shape and the size of soft object $f_i(X)=0.5$. For example, an ellipsoid $d_i(X)=(x/a)^2+(y/b)^2+ (z/c)^2=1$ is used as an influential region to define $d_i(X)$. Thus, the shape $(P \circ d_i)(X)=0.5$ is similar to the ellipsoid, as shown in Fig. 1.

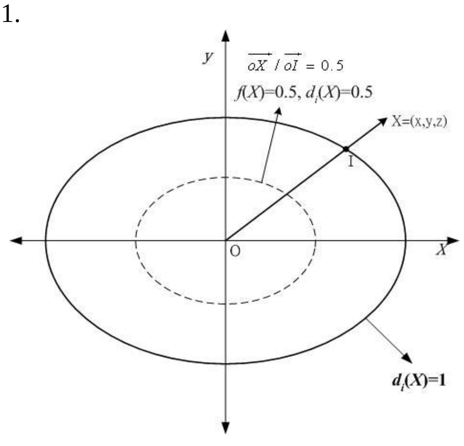


Figure 1. Influential region $d_i(X)=1$, solid line, and the shape of soft object $f_i(X)=(P \circ d_i)(X)=0.5$, i.e. $d_i(X)=0.5$, dotted line.

Some distance functions are introduced as follows:

- Super-Ellipsoids [11]:

$$d(x, y, z)=((|x/a|^{p1}+|y/b|^{p1}+|z/c|^{p1})^{1/p1} \quad (1)$$

- Super-Quadrics or super-ellipsoids [11]:

$$d(x,y,z)=((|x/a|^{p_1}+|y/b|^{p_1})^{p_2/p_1}+|z/c|^{p_2})^{1/p_2} \quad (2)$$

- Generalized distance functions [12]:

$$d(x,y,z)=(\textstyle\sum_{i=1}^{k} |[x,y,z] \bullet \boldsymbol{n}_i|^p)^{1/p} \quad (3)$$

where p_2 and p_1 are required to be larger than 1 and symbol $\bullet$ represents dot product and $\boldsymbol{n}_i$, $i=1, \ldots, k$, are unit normal vectors of planes.

B. Blending Operations

Blending operations play an important role in creating a complex soft object. A blending operation smoothly connects k primitive soft objects $f_1(X)=0.5$, ..., and $f_k(X)=0.5$ via a blending operator $B_k(x_1, ..., x_k):R_+^k \rightarrow R_+$ and is defined by the point set:

$$\{X \in R^3 | B_k(f_1(X),...,f_k(X))=0.5\}$$

Existing blending operations provided union, intersection and difference operations [1]-[8], [10]. Ref. [1], [5]-[7] discussed about blending operations with blend range control, Ref. [10] introduced soft blend by:

$$B_k(x_1,\ldots,x_k)= x_1+x_2+\ldots+x_k$$

And on the basis of *Max* and *Min* operations Ref. [8] presented super-ellipsoidal:

$$\text{Union: } B_k(x_1,\ldots,x_k)=(x_1^p+\ldots+x_k^p)^{1/p}$$

$$\text{Intersection: } B_k(x_1,\ldots,x_k)=(x_1^{-p}+\ldots+x_k^{-p})^{-1/p}$$

where p determines the softness of the blending.

Fig. 2 shows a die created by a difference operation of a cube from 21 spheres and the cube is an intersection of 3 pairs of parallel planes.

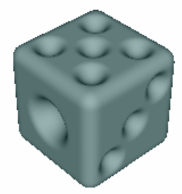


Figure 2. A die generated from a difference operation of a cube from 21 spheres.

C. Ray-Linear Functions

Since a distance function needs to be represented in the form of influential radius, this section presents sufficient conditions for a function to fulfill this requirement.

Definition 1 [16]**:** $f(X):R^3 \rightarrow R_+$ is called non-negative ray-linear if $f(aX)=af(X)$ holds for any $X \in R^3$ and $a \in R_+$.

For short, "ray-linear" stands for "non-negative ray-linear". **Theorem 1** offers a sufficient condition for a function to be represented using influential radius [6].

Theorem 1: If $f(X):R^n \rightarrow R_+$ is ray-linear, then $f(X)$ can be reformulated to be r/R_f, where $r=\|X\|$ and R_f is the influential radius of $f(X)=1$ for X, and hence $f(X)$ is qualified to be a distance function.

From **Theorem** 1 (1)-(3) can be shown ray-linear. Besides, **Theorem 2** also shows how a blending operation $B_k(f_1(X), ..., f_k(X))$ is qualified to be a distance function as follows:

Theorem 2: If all $f_i(X)$, i=1, ..., k, are ray-linear and $B_k(x_1, ..., x_k)$:$R_+^k \rightarrow R_+$ is ray-linear, then $B_k(f_1(X), ..., f_k(X))$ is ray-linear and is qualified to be a distance function.

D. Affine Transformation

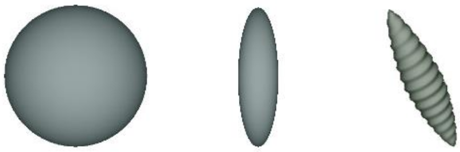
Let M be a 4×3 matrix written by:

$$M=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix}$$

And P be [x, y, z, 1] where [x, y, z]=X. Then, an affine transformation on a soft object $f_i(X)$=0.5 by matrix M is defined by:

$$f_i(A(X))= P(d_i(A(X)))=0.5 \text{ and } A(X)=PM$$

As shown in Fig. 3, in the matrix M, the first three rows produce rotation, scaling and shearing, and the last row translation; bedsides if the elements a, …, l of M are defined using position functions, then twisting and tendering can be obtained [13], [14].



(a) (b) (c)

Figure 3. (a) A sphere. (b) A sphere after scaling with respect to x-axis. (c) A sphere after twisting with respect to z-axis and scaling.

III. Erosion and Dialation Operations

Unlike affine transform, erosion and dilation operations were developed in [15] to deform a soft object by distance function operations. But they enlarge or shrink a soft object totally as described in this section.

A. Erosion Operation of Distance Functions

Let $d(X)$ and $d_E(X)$ be ray-linear distance functions, and let $d(X)$=1 and $d_E(X)$=1 be closed surfaces, satisfying $\{X \in R^3 | d(X)=1\} \subset \{X \in R^3 | d_E(X)=1\}$. Then, an erosion operation of $d(X)$=1 by $d_E(X)$=1, where $d_E(X)$ called an eroding function, is defined as:

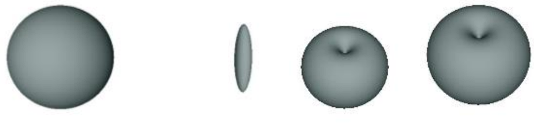
$$B_E(d(X), d_E(X))=(d(X)^{-n} - d_E(X)^{-n})^{1/-n} \quad (4)$$

And $n \geq 1$. Because (4) is ray-linear, it is reformulated by substituting $d(X)$ with r/R_d and $d_e(X)$ with r/R_{de} to be:

$$B_E(d(X), d_E(X))=r/R_{BE}= r/(R_d^{\,n} - R_{dE}^{\,n})^{1/n} \quad (5)$$

It follows from (5) that when n=1, $B_{E2}(d(X), d_E(X)) =r/(R_d-R_{dE})$, which implies (4) reduces the influential radius R_d of $d(X)$ to (R_d-R_{dE}). As a result, the erosion $B_E(d(X), d_E(X))$=1 is like the surface $d(X)$=1 on which every point (x, y, z) is moved R_{dE} toward the origin. That is, it is viewed as the object $d(X)$=1 eroded by the influential radius R_{dE} of $d_E(X)$=1. For example, let $d(X)$ be $(|x/30|^2+|y/30|^2+|z/30|^2)^{1/2}$ and $d_E(X)$ be a super- ellipsoid $(|x/5|^2+|y/5|^2+|z/20|^2)^{1/2}$. The erosion $B_E(d, d_E)(X)$ =1, n=1, in Fig. 4(c) can be viewed as the sphere in Fig. 4(a) eroded by the object in Fig. 4(b).

However, erosion operation faces a problem that it always shrink the whole surface $d_E(X)$=1, as shown in Fig. 4(c). It is impossible for erosion operation to generate an object like that in Fig. 4(d), which has similar size with $d(X)$=1 in Fig. 4(a) and is created by the proposed operations stated in Section IV.



(a) (b) (c) (d)

Figure 4. (a) $d(X)$=1: a sphere. (b) $d_E(X)$=1: an ellipsoid. (c) Erosion of the sphere in (a) by the ellipsoid in (b). (d) Partial erosion of the object in (c), which has a similar size with the sphere in (a).

B. Dilation Operation of Distance Functions

Let $d(X)$ and $d_D(X)$ be ray-linear distance functions, and let $d(X)$=1 and $d_D(X)$=1 be closed surfaces, satisfying $\{X \in R^3 | d(X)=1\} \subset \{X \in R^3 | d_D(X)=1\}$. Then, an dilation operation of $d(X)$=1 by $d_D(X)$=1, where $d_D(X)$ called an dilating function, is defined as:

$$B_D(d(X), d_D(X))=(d(X)^{-n} + d_D(X)^{-n})^{1/-n} \quad (6)$$

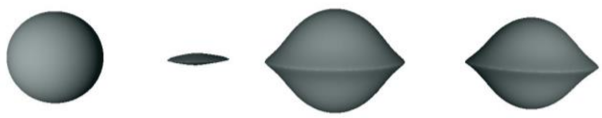
And $n \geq 1$. Because (6) is ray-linear, it is reformulated by substituting $d(X)$ with r/R_d and $d_D(X)$ with r/R_{dD} to be

$$B_D(d(X), d_D(X))=r/R_{BD}= r/(R_d^{\,n} + R_{dD}^{\,n})^{1/n} \quad (7)$$

From (7), it is obtained that when n=1, $B_D(d(X), d_D(X))=r/(R_d+R_{dD})$, which means that (6) increases the influential radius R_d of $d(X)$ to (R_d+R_{dD}). Therefore, the dilation $B_D(d(X), d_D(X))$=1 is like the surface $d(X)$=1 on which every point (x, y, z) is moved R_{de} away from the origin. That is, it is viewed as the object $d(X)$=1 dilated by the influential radius R_{dD} of $d_D(X)$=1.

For example, let $d(X)=(|x/30|^{20}+|y/30|^{20}+|z/30|^{20})^{1/20}$ and $d_D(X)=((|x/15|^2+|y/15|^2)^{1.25/2}+|z/4|^{1.25})^{1/1.25}$ and the shapes $d(X)$=1 and $d_D(X)$=1 are shown in Fig. 5(a)-(b). Thus, the dilation $B_D(d, d_D)(X)$=1, n=1, generates a UFO-shaped object in Fig. 5(c).

Similar to erosion operations, dilation operation faces a problem that it always enlarge the whole surface $d_D(X)$=1, too, as shown in Fig. 5(c). It is impossible for dilation operation to generate an object like that in Fig. 5(d), which has a similar size with that in Fig. 5(a) and is created by the proposed operations stated in Section IV.



(a) (b) (c) (d)

Figure 5. (a) $d(X)$=1: a sphere. (b) $d_D(X)$=1: a disk. (c) Dilation of the sphere in (a) by the disk in (b). (d) Partial dilation of the object in (c), which has a similar size with the sphere in (a).

IV. Generalized Erosion and Dilation Operations

To conquer the problems of erosion and dilation operations stated in Section III and obtain partial erosion and partial dilation of a soft object for local deformation, this paper proposes generalized erosion and dilation operations of distance functions.

Let $d(X)$ and $d_{*i}(X)$, i=1, ..., k, be ray-linear distance functions, and let $d(X)$=1 and $d_{*i}(X)$=1 be closed surfaces, satisfying $\{X\in R^3|d(X)=1\}\subset\{X\in R^3|d_{*i}(X)=1\}$. Then, a generalized erosion and dilation operation of distance functions is defined as:

$$B_{Gk}(d(X), d_{*1}(X), ..., d_{*k}(X))= (d(X)^{-n} \pm d_{*1}(X)^{-n}\pm d_{*2}(X)^{-n}\pm...\pm d_{*k}(X)^{-n})^{1/-n} \quad (8)$$

where symbol $\pm$ stands for an addition or a subtraction operator and $d_{*i}(X)$ denotes an eroding function $d_E(X)$ or a dilating function $d_D(X)$.

Equation (8) is shown ray-linear below: for any $a\in R_+$, the following always hold:

$B_{Gk}(d(aX), d_{*1}(aX), ..., d_{*k}(aX))=$
$(d(aX)^{-n} \pm d_{*1}(aX)^{-n}\pm d_{*2}(aX)^{-n}\pm...\pm d_{*k}(aX)^{-n})^{1/-n} =$
$(a^{-n}d(X)^{-n} \pm a^{-n}d_{*1}(X)^{-n}\pm a^{-n}d_{*2}(X)^{-n}\pm...\pm a^{-n}d_{*k}(X)^{-n})^{1/-n} =$
$a\,(d(X)^{-n} \pm d_{*1}(X)^{-n}\pm d_{*2}(X)^{-n}\pm...\pm d_{*k}(X)^{-n})^{1/-n}=$
$a\,B_{Gk}(d(X), d_{*1}(X), ..., d_{*k}(X))$.

The last equation completes the proof. Due to the ray-linear property, it follows from Theorem 1 that:

- Equation (8) is qualified to be a distance function for defining a soft object by:

$$(P\circ B_{Gk}(d(X), d_{*1}(X),..., d_{*k}(X)))=0.5$$

- Equation (8) is reformulated by substituting $d(X)$ with r/R_d and $d_*i(X)$ with r/R_{d*i} to be:

$$B_{Gk}(d(X), d_{*1}(X),..., d_{*k}(X))=r/R_{BGk}= r/(R_d^{\,n} \pm R_{d*1}^{\,n}\pm R_{d*2}^{\,n}\pm... \pm R_{d*k}^{\,n})^{1/n} \quad (9)$$

Equations (8) and (9) are analyzed as follows:

(1). When n=1, $B_{Gk}=r/(R_d\pm R_{d*1}\pm R_{d*2}\pm...\pm R_{d*k})$, which implies that (9) sequentially and repeatedly increases and decreases the influential radius R_d of $d(X)$ to $(R_d \pm R_{d*1}\pm R_{d*2}\pm...\pm R_{d*k})$. Therefore, the surface $B_{Gk}(X)$=1 is like the surface $d(X)$=1 on which every point (x, y, z) is moved R_{d*i}, i=1, ..., k, away from or toward the origin. In other word, the surface $B_{Gk}(X)$=1 is like $d(X)$=1 eroded by $d_{Di}(X)$=1 and dilated by $d_{Ei}(X)$=1 sequentially and repeatedly; soft object $(P\circ B_{Gk}(X))$=0.5 is like the surface $d(X)$=0.5 eroded by $d_{Di}(X)$=0.5 and dilated by $d_{Ei}(X)$=0.5 sequentially and repeatedly.

(2). As n increases from 1 to ∞, the influential radius R_{BGk} of $B_{Gk}(X)$=1 varies from $(R_d \pm R_{d*1}\pm R_{d*2}\pm...\pm R_{d*k})$ to R_d. Consequently, the dilation and erosion effect caused by $d_{*i}(X)$, i=1, ..., k, decreases as n increases. Hence, the surface $B_{Gk}(X)$=1 restore to the original surface $d(X)$=1 and soft object $(P\circ B_{Gk}(X))$=0.5 restore to the surface $d(X)$=0.5 before deformation, as n increases from 1 to ∞.

Two special cases of B_{Gk} in (8) are discussed below:

- Multiple partial erosions:

$$B_{Gk}(d(X), d_{E1}(X), d_{D2}(X), d_{E3}(X), d_{D4}(X),\ldots)= (d(X)^{-n}-d_{E1}(X)^{-n}+d_{D2}(X)^{-n}-d_{E3}(X)^{-n}+d_{D4}(X)^{-n}+...)^{1/-n} \quad (10)$$

Equation (10) offers partial erosion, double partial erosion or …, enabling erosion operation in (4) to deform $d(X)$=1 almost locally. In (10), $d_{E1}(X)$, and $d_{E3}(X)$,… are eroding functions, but $d_{D2}(X)$ is a restoring (dilating) function with respect to $d_{E1}(X)$ to restore $B_{G1}(d(X), d_{E1}(X))$=1 to the original size of $d(X)$=1, and $d_{D4}(X)$ with respect to $d_{E3}(X)$ to restore $B_{G3}(d(X), d_{E1}(X), d_{D2}(X), d_{E3}(X))$=1 to $B_{G2}(d(X), d_{E1}(X), d_{D2}(X))$=1, and the rest are the same.

- Multiple partial dilations:

$$B_{Gk}(d(X), d_{D1}(X), d_{E2}(X),d_{D3}(X), d_{E4}(X),\ldots)= (d(X)^{-n}-d_{D1}(X)^{-n}+d_{E2}(X)^{-n}-d_{D3}(X)^{-n}+d_{E4}(X)^{-n}+...)^{1/-n} \quad (11)$$

Equation (11) offers partial dilation, double partial dilation or …, enabling dilation operation in (6) to deform $d(X)$=1 almost locally. In (11), $d_{D1}(X)$, $d_{D3}(X)$, and … are dilating functions, but $d_{E2}(X)$ is a restoring (eroding) function with respect to $d_{D1}(X)$ to restore $B_{G1}(d(X), d_{D1}(X))$=1 to the original size of $d(X)$=1, and $d_{E4}(X)$ with respect to $d_{D3}(X)$ to restore $B_{G3}(d(X), d_{D1}(X), d_{E2}(X), d_{D3}(X))$=1 to $B_{G2}(d(X), d_{D1}(X), d_{E2}(X))$=1, and the others are the same.

V. Deforming Soft Objects by Generalized Erosion and Dilation Operations

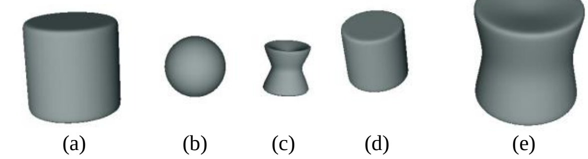
This section demonstrates some deformation effects on soft objects by using the proposed generalized erosion and dilation operations in (8), (10) and (11).

A. Concave Effect

Let $d(X)$ be a cube $(|x/30|^{20}+|y/30|^{20}+|z/30|^{20})^{1/20}$ and $d_{E1}(X)$ a sphere $(|x/20|^2+|y/20|^2+|z/20|^2)^{1/2}$. As in Fig. 6(c), the erosion of $B_{G1}(d, d_{E1})(X)$=1, n=1, makes the cube $d(X)$=1 become concave by erosion of a sphere, but it shrinks $d(X)$=1 totally. This is solved by partial erosion $B_{G2}(d(X), d_{E1}(X), d_{D2}(X))$=1 with restoring function $d_{D2}(X)$ $=(|x/20|^{20} +y/20|^{20}+|z/20\ |^{20})^{1/20}$ in Fig. 6(d), which makes the size of $B_{G1}(d,d_{E1})(X)$=1 become more similar to that of the cube in Fig. 6(a).

(a) (b) (c) (d) (e)

Figure 6. (a) $d(X)$=1: a cube. (b) $d_{E1}(X)$ =1: a sphere. (c) Concave cube, obtained from the erosion of the cube in (a) by the sphere in (b). (d) $d_{D2}(X)$=1: restoring function of a cube. (e) Partial erosion of the concave object in (c) by using the cube in (d) as a restoring function.



(a) (b) (c) (d) (e)

Figure 7. (a) $d(X)$=1: a cylinder. (b) $d_{E1}(X)$=1: a sphere. (c) Concave cylinder, obtained from the erosion of the cube in (a) by the sphere in (b). (d) $d_{D2}(X)$=1: a restoring function of a cylinder. (e) Partial erosion of the concave object in (c) using the cylinder in (d) as a restoring function.

Let $d(X)$ be a cylinder $((|x/30|^2+|y/30|^2)^{20/2}+|z/30|^{20})^{1/20}$ and $d_{E1}(X)$ a sphere $(|x/20|^2+|y/20|^2+|z/20|^2)^{1/2}$. Erosion of

$B_{G1}(d, d_{E1})(X)=1$, $n=1$, in Fig. 7(c) shows that the cube $d(X)=1$ becomes concave due to the erosion by $d_{E1}(X)$. Since the erosion in Fig. 7(c) shrinks $d(X)=1$ totally, this is solved by partial erosion $B_{G2}(d(X),d_{E1}(X),d_{D2}(X))=1$ with restoring function $d_{D2}(X)=((|x/20|^2+|y/20|^2)^{20/2}+|z/20|^{20})^{1/20}$ in Fig. 7(d), which makes the size of $B_{G1}(d, d_{E1})(X)=1$ become more like that of the cube in Fig. 7(a).

B. Convex Effect

Let $d(X)$ be a cube $(|x/30|^{20}+|y/30|^{20}+|z/30|^{20})^{1/20}$ and $d_{D1}(X)$ a sphere $(|x/20|^2+|y/20|^2+|z/20|^2)^{1/2}$. Dilation operation $B_{G1}(d, d_{D1})(X)=1$, $n=1$, as in Fig. 8(c), makes the cube $d(X)$ become convex due to the dilation by a sphere, but it also enlarges $d(X)=1$ totally. However, partial dilation $B_{G2}(d(X), d_{D1}(X), d_{E2}(X))=1$ with restoring function $d_{E2}(X)=(|x/20|^{20}+|y/20|^{20}+|z/20|^{20})^{1/20}$ in Fig. 8(d) makes the size of $B_{G1}(d, d_{D1})(X)=1$ become more similar to that of the cube in Fig. 8(a).

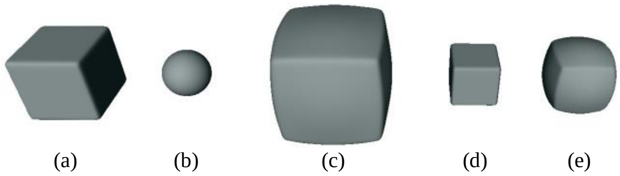


Figure 8. (a) $d(X)=1$: a cylinder. (b) $d_{D1}(X)=1$: a sphere. (c) Convex cube, obtained from the dilation of the cube in (a) by the sphere in (b). (d) $d_{E2}(X)=1$: a restoring function of a cube. (e) Partial erosion of the object in (c) by using the cube in (d) as a restoring function.

C. Point Erosion

Let $d(X)$ be a sphere $(|x/30|^2+|y/30|^2+|z/30|^2)^{1/2}$, $d_{E1}(X)$ be a ellipsoid $(|x/5|^2+|y/5|^2+|z/20|^2)^{1/2}$, $d_{E3}(X)$ be a ellipsoid $(|x/5|^2+|y/20|^2+|z/5|^2)^{1/2}$, and $d_{D2}(X)$ and $d_{D4}(X)$ be spheres $(|x/5|^2+|y/5|^2+|z/5|^2)^{1/2}$. As in Fig. 9(d), $B_{G2}(d(X), d_{E1}(X), d_{E3}(X))=1$, $n=1$, creates four holes on the sphere $d(X)=1$ due to double erosions by ellipsoids $d_{E1}(X)$ and $d_{E3}(X)$, but it shrinks $d(X)=1$ totally. This is solved by double partial erosion $B_{G3}(d(X), d_{E1}(X), d_{D2}(X), d_{E3}(X), d_{D4}(X))=1$ with restoring function $d_{D2}(X)$ and $d_{D4}(X)$ in Fig. 9(e), which makes the size of $B_{G2}=1$ become more similar to that of the sphere in Fig. 9(a).

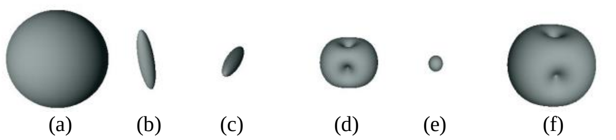


Figure 9. (a) $d(X)=1$: a sphere. (b)-(c) $d_{E1}(X)=1$ and $d_{E3}(X)=1$: ellipsoids. (d) Sphere with four holes, obtained from the erosion of the sphere in (a) by the ellipsoids in (b)-(c). (e) $d_{D2}(X)=1$ and $d_{D4}(X)=1$: restoring functions of a sphere. (f) Double partial erosion of the object in (d) using the sphere in (e) as restoring functions.

D. Point Dilation

Let $d(X)$ be a sphere $(|x/30|^2+|y/30|^2+|z/30|^2)^{1/2}$, $d_{D1}(X)$ and $d_{D3}(X)$ be $(|x/5|^2+|y/5|^2+[z/15,0]^2+[-z/5,0]^2)^{1/2}$ where [*,0] means * for *>0, otherwise 0, and $d_{E2}(X)$ and $d_{E4}(X)$ be spheres $(|x/5|^2+|y/5|^2+|z/5|^2)^{1/2}$. As in Fig. 10(c), dilation operation $B_{G1}(d(X), d_{D1}(X))=1$, $n=1$, make the sphere generate a pointed end, but it shrinks $d(X)=1$ totally. This is solved by partial erosion $B_{G3}(d(X), d_{D1}(X), d_{E2}(X))=1$ and double partial erosion $B_{G3}(d(X), d_{D1}(X), d_{E2}(X), d_{D3}(X), d_{E4}(X))=1$ with restoring functions $d_{E2}(X)$ and $d_{E4}(X)$ in Fig. 9(d), which makes the size of $B_{G2}=1$ more similar to that of the sphere $d(X)=1$ in Fig. 10(a).

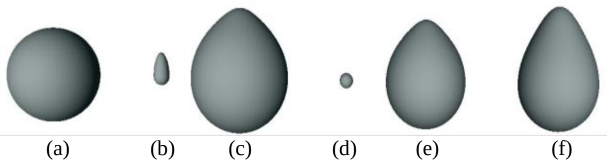


Figure 10. (a) $d(X)=1$: a sphere. (b) $d_{D1}(X)=1$ and $d_{D3}(X)=1$: ellipsoids. (c) Sphere with a pointed end obtained from the dilation of the sphere in (a) by the object in (b). (d) $d_{E2}(X)=1$ and $d_{E4}(X)=1$: restoring functions of a sphere. (e) Partial erosion of the object in (c) using the sphere in (d) as restoring function. (f) Double partial erosion of the object in (c) using the sphere in (d) as restoring functions.

E. Disk Erosion

Let $d(X)$ be a cube $(|x/30|^{10}+|y/30|^{10}+|z/30|^{10})^{1/10}$ and $d_{E1}(X)$ a disk $((|x/20|^{20}+|y/20|^{10})^{2/10}+|z/20|^2)^{1/2}$. The erosion of $B_{G1}(d(X), d_{E1}(X))=1$, $n=1$, is shown in Fig. 11(c), and partial erosion $B_{G2}(d(X), d_{E1}(X), d_{D2}(X))=1$ with restoring function $d_{D2}(X)=(|x/5|^{10}+|y/5|^{10}+|z/5|^{10})^{1/10}$ is in Fig. 11(d) making the size of $B_{G1}(d(X), d_{E1}(X))(X)=1$ more similar to that of the cylinder in Fig. 11(a).

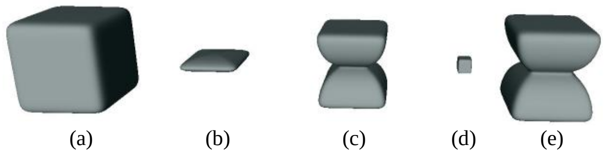


Figure 11. (a) $d(X)=1$: a cube. (b) $d_{E1}(X)=1$: a cubic disk. (c) Erosion of the cube in (a) by the disk in (b). (d) $d_{D2}(X)=1$: a restoring function of a cube. (e) Partial erosion of the object in (c) using the cube in (d) as a restoring function.

F. Disk Dilation

Let $d(X)$ be a sphere $(|x/30|^2+|y/30|^2+|z/30|^2)^{1/2}$, $d_{D1}(X)$ and $d_{D3}(X)$ be disks $((|x/20|^2+|y/20|^2)^{1.25/2}+|z/4|^{1.25})^{1.25}$, and $d_{E2}(X)$ and $d_{E4}(X)$ be spheres $(|x/4|^2+|y/4|^2+|z/4|^2)^{1/2}$. As in Fig. 12(c), dilation $B_{G1}(d(X), d_{D1}(X))=1$, $n=1$, creates a UFO-shaped object, but it shrinks $d(X)=1$ totally. This is solved by partial dilation $B_{G2}(d(X), d_{D1}(X), d_{E2}(X))=1$ and double partial dilation $B_{G4}(d(X), d_{D1}(X), d_{E2}(X), d_{D3}(X), d_{E4}(X))=1$ with restoring functions $d_{E2}(X)$ and $d_{E4}(X)$ in Fig. 12(d), which restore the size of dilation $B_{G1}=1$ back to the original size of the sphere $d(X)=1$ in Fig. 12(a).

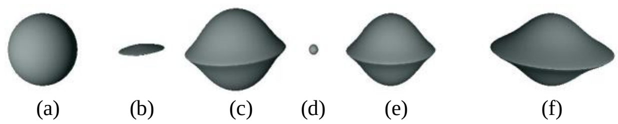


Figure 12. (a) $d(X)=1$: a sphere. (b) $d_{D1}(X)=1$ and $d_{D3}(X)=1$: disk. (c) UFO-shaped object, obtained from the dilation of the sphere in (a) by the disk in (b). (d) $d_{E2}(X)=1$ and $d_{E4}(X)=1$: restoring functions of a sphere. (e) Partial erosion of the object in (c) using the sphere in (d) as restoring functions. (f) Double partial erosion of the object in (c) using the sphere in (d) as restoring functions.

In addition, as parameter n increases from 1, 2, 4 to 8, the shape of double partial dilation $B_{G4}(d(X), d_{D1}(X), d_{E2}(X), d_{D3}(X), d_{E4}(X))=1$ varies and finally restores to the original sphere $d(X)=1$, as shown from the left to right objects in Fig. 13(b).

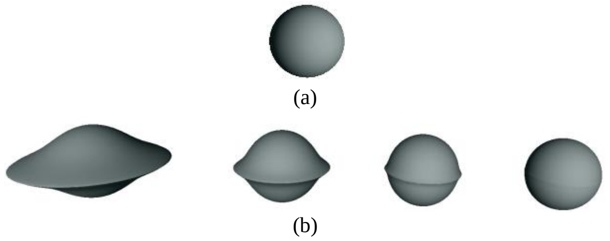


Figure 13. (a) $d(X)$=1: the original sphere. (b) UFO-shaped objects: double partial dilations $B_{G4}(d(X), d_{D1}(X), d_{E2}(X), d_{D3}(X), d_{E4}(X))$=1 with parameter n increased from 1, 2, 4 to 8 for the objects from left to right, where the rightmost object is similart to the sphere in (a).

G. Four-Balled Shape

Let $d(X)$ be a sphere $(|x/30|^{20}+|y/30|^{20}+|z/30|^{20})^{1/20}$ and $d_{E1}(X)$ be $((|x/15|^{20}+|y/15|^{20})^{2/20}+|z/20|^2)^{1/2}$. As in Fig. 14(c), erosion operation $B_{G1}(d, d_{E1})(X)$=1, n=1 makes the sphere $d(X)$=1 become a four-balled shape, but it also shrink $d(X)$=1 totally. However, partial dilation $B_{G2}(d(X), d_{E1}(X), d_{D2}(X))$=1 with restoring function $d_{D2}(X)=((|x/15|^2+|y/15|^2+|z/15|^2)^{1/2}$ in Fig. 14(d) makes the size of $B_{G1}(d, d_{E1})(X)$=1 become more similar to that of the sphere $d(X)$=1 in Fig. 14(a).

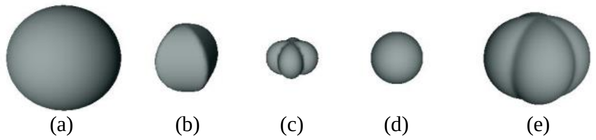


Figure 14. (a) $d(X)$=1: a sphere. (b) $d_{E1}(X)$=1: a cubic ellipsoid. (c) A four-balled shape, obtained from the erosion of the sphere in (a) by the object in (b). (d) $d_{D2}(X)$=1: restoring function of a sphere. (e) Partial erosion of the object in (c) by using the cube in (d) as a restoring function.

In addition, as parameter n increases from 1, 2, 4 to 8, thr shape of partial erosion $B_{G2}(d(X), d_{E1}(X), d_{D2}(X))$=1 varies and finally restores to the original sphere $d(X)$=1, as shown from the left to right objects in Fig. 15(b).

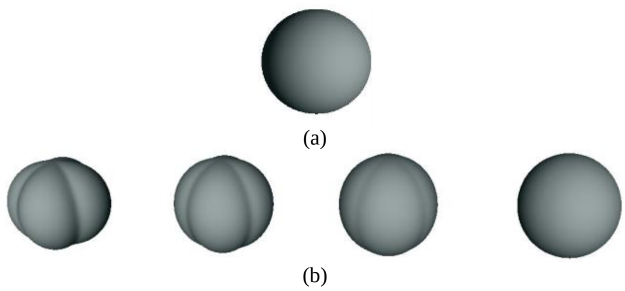


Figure 15. (a) $d(X)$=1: the original sphere. (b) Four-balled objects: partial erosions $B_{G2}(d(X), d_{E1}(X), d_{D2}(X))$=1 with parameter n increased from 1, 2, 4 to 8 for the objects from left to right, where the rightmost object is similart to the sphere in (a).

H. Six-Balled Shape

Let $d(X)$ be a sphere $(|x/30|^{20}+|y/30|^{20}+|z/30|^{20})^{1/20}$ and $d_{E1}(X)$ be $((|x/15|^{10}+|y/15|^{10}+|z/15|^{10})^{1/10}$. As in Fig. 16(c), erosion $B_{G1}(d(X), d_{E1}(X))$=1, n=1 makes the sphere $d(X)$=1 become a six-balled shape, but it also shrink $d(X)$=1 totally. However, partial dilation $B_{G2}(d(X), d_{E1}(X), d_{D2}(X))$=1 with restoring function $d_{D2}(X)=((|x/15|^2+|y/15|^2+|z/15|^2)^{1/2}$ in Fig. 16(d) makes the size of $B_{G1}(d(X), d_{E1}(X))$=1 become more similar to that of the sphere in Fig. 16(a).

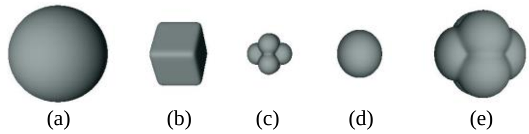


Figure 16. (a) $d(X)$=1: a sphere. (b) $d_{D1}(X)$=1: a cube. (c) Six-balled object, obtained from the erosion of the sphere in (a) by the cube in (b). (d) $d_{E2}(X)$=1: restoring function of a sphere. (e) Partial erosion of the object in (c) using the sphere in (d) as a restoring function.

In addition, as parameter n increases from 1, 2, 4 to 8, thr shape of partial dilation $B_{G2}(d(X), d_{D1}(X), d_{E2}(X))$=1 changes and finally restores to the original sphere $d(X)$=1, as shown from the left to right object in Fig. 17(b).

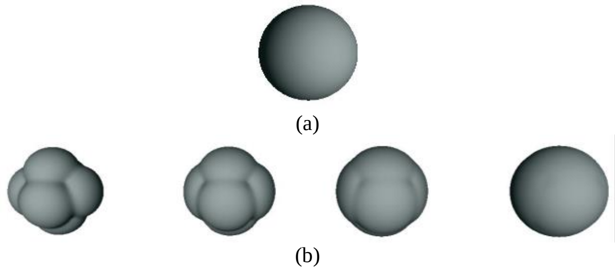


Figure 17. (a) $d(X)$=1: the original sphere. (b) Six-balled objects: partial dilations $B_{G2}(d(X), d_{D1}(X), d_{E2}(X))$=1 with parameter n increased from 1, 2, 4 to 8 for the objects from left to right, where the rightmost object is similar to the sphere in (a).

VI. Conclusion

In soft object modeling, to enable the shapes of primitive soft objects to become more diverse this paper has proposed generalized erosion and dilation operations of distance functions. Unlike traditional method of affine transform which deforms an object by coordinate transformation, the proposed operations deform a primitive soft object by replacing its distance function with a distance function operation composed of the distance function defining the soft object and a sequence of, more than two, chosen eroding and dilating functions. In the distance function operation, dilating function is used to increase the influential radius of the soft object and eroding function to decrease the influential radius. As a result, the chosen eroding and dilating functions can dilate and erode a primitive soft object successively and sequentially by varying the influential radius of the soft object. Compared to other existing deformation method:

- The proposed operations erode (shrunk) and dilate (enlarge) a soft object sequentially and repeatedly through a sequence of freely chosen eroding and dilating distance functions. Consequently, double, triple, and multiple erosions and dilations are also allowed whereas existing erosion and dilation operations allows only a single erosion or dilation operation.
- The proposed operations allow a soft object to be eroded and dilated locally, not totally, via restoring functions. That is, partial erosion and partial dilation, and even double, triple and

multiple partial erosions and partial dilations are allowed while existing erosion and dilation operations cannot do these.

- The proposed operations can deform a soft object to create a totally different shape. For example, a sphere becomes disk-shaped or four-ball-shaped after deformation by the proposed operations. However, affine transform usually keeps a similar shape after deformation.

On the other hand, on the basis of the proposed operations, many new deformation effects have been created such as concave, convex, point erosion, disk erosion, point dilation, disk dilation, and disk-shaped apple-shaped, two-ball-shaped, four-ball-shaped, six-ball-shaped and UFO-shaped objects.

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REFERENCES

[1] L. Barthe, B. Wyvill, and E. D. Groot, "Controllable binary CSG operators for soft objects," *International Journal of Shape Modeling*, vol. 10, no. 2, pp. 133-154, 2004.

[2] A. Bernhardt, L. Barthe, M. P. Cani, and B. Wyvill, "Implicit blending revisited," *Computer Graphics Forum*, vol. 29, no. 2, pp. 367-375, 2010.

[3] O. Gourmel, L. Barthe, M. Cani, B. Wyvill, A. Bernhardt, M. Paulin, and H. Grasberger, "Gradient-Based implicit blend," *ACM Trans. on Graphics*, vol. 32, no. 2, pp. 1-12, 2013.

[4] E. Groot, B. Wyvill, and H. Wetering, "Locally restricted blending of Blobtrees," *Computers & Graphics*, vol. 33, no. 6, pp. 690-697, 2009.

[5] Q. Li, "Smooth piecewise polynomial blending operations for implicit shapes," *Computer Graphics Forum*, vol. 26, no. 2, pp. 157-171, 2007.

[6] P. C. Hsu and C. Lee, "The scale method for blending operations in functionally based constructive geometry," *Computer Graphics Forum*, vol. 22, no. 2, pp. 143-158, 2003.

[7] G. I. Pasko, A. A. Pasko, and T. L. Kunii, "Bounded blending for function-based shape modeling," *IEEE Computer Graphics and Application*, vol. 25, no. 2, pp. 36-44, 2005.

[8] A. Ricii, "A constructive geometry for computer graphics," *The Computer Journal*, vol. 16, no. 2, pp. 157-160, May 1973.

[9] P. C. Hsu and C. Lee, "Field functions for blending range controls on soft objects," *Computer Graphics Forum*, vol. 22, no. 3, pp. 233-242, 2003.

[10] G. Wyvill, C. McPheeters, and B. Wyvill, "Data structure for soft objects," *The Visual Computer*, vol. 2, no. 4, pp. 227-234, 1986.

[11] A. H. Barr, "Super-Quadrics," *IEEE Computer Graphics and Applications*, vol. 1, no. 1, pp. 11-23, 1981.

[12] E. Akleman and J. Chen, "Generalized distance functions," in *Proc. International Conference on Shape Modeling and Application*, 1999, pp. 72-79.

[13] A. Barr, "Global and local deformations of solid primitives," *SIGGRAPH Computer Graphics*, vol. 18, no. 3, pp. 21-30, 1984.

[14] B. Wyvill, A. Guy, and E. Galin, "Extending the CSG tree: Warping, blending and Boolean operations in an implicit surface modeling systems," in *Proc. Implicit Surfaces'98*, 1998, pp. 128-136.

[15] P. C. Hsu, "Distance function operations for developing new field functions in soft object modeling," *International Journal of Computer and Communication Engineering*, vol. 2, no. 2, pp. 179-183, Mar. 2013.

[16] E. Akleman, "Interactive construction of smoothly blended star solids," in *Proc. Graphical Interface'96*, May 1996.

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