Orthogonal Vector Interpolation for Aesthetic Image Transformations

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Abstract—**In this paper we present and examine a general SVD-based mathematical framework for the manipulation and transformation of images for traditional operations such as image compression and super-resolution to computational photography applications, e.g., to achieve novel aesthetic effects for the rapidly growing social media demand for image filter apps. Examples are given to demonstrate how images can be perturbed from realistic toward fanciful or from airbrushed gloss toward gritty realism. A primary goal of this paper is to draw greater attention to algorithmic methods that go beyond the traditional parameters of conventional image manipulation.**

Index Terms—**computational photography, singular value decomposition, SVD, orthogonal vector interpolation, OVI, image processing, image enhancement, image compression, image super-resolution**

I. Introduction

In this paper we introduce Orthogonal Vector Interpolation (OVI) as a mathematical methodology for the “holistic” manipulation and transformation of images to achieve various aesthetic effects. This is a rapidly emerging topic of interest in social media driven by the growing demand for image filter apps, e.g., on Instagram and Snapchat.

The structure of the paper is as follows: In Section 2 we introduce the SVD-based OVI framework and give an example of its applicability to the traditional problems of image decimation and super-resolution. In Section 3 we demonstrate its more general applicability for creating novel aesthetic transformations. And Section 4 concludes with a brief discussion and summary.

II. Orthogonal Vector Interpolation (OVI)

For any $m \times n$ matrix M there exists a singular value decomposition (SVD)

$$M = UDV^T \quad (1)$$

where U is an $m \times k$ matrix with orthonormal columns, D is a $k \times k$ nonnegative diagonal matrix, and V is a $k \times n$ with orthonormal columns. The non-increasing set of ordered positive values $\sigma_i^k = D_{ii}$ are referred to as the singular values of M, the columns of U and V are referred to as the singular vectors of M, and the rows *and* columns are jointly referred to as the orthogonal vectors of M.

The importance of the SVD is that it decomposes a matrix (e.g., an image) into a weighted sum of r =rank(M) rank-1 matrices $U_i\sigma_i V_i^T$, $0 \leq i \leq r$. In other words, it determines an k-dimensional orthogonal coordinate frame in which the content of the matrix can be expressed as a sum of k coordinate projections. Although the orientation of this coordinate frame is not unique, the weights $\sigma_1 \ldots \sigma_k$ are invariant i.e., the amount of information associated with the different projections is an intrinsic property of the matrix. When the matrix represents intensity/color values for pixels of an image then the set of singular values represents a signature that is invariant with respect to a variety of natural image transformations, e.g., rotations, while the orthogonal left and right singular vectors encode the structural information that distinguishes a given image from all other images having the same signature.

The attractive mathematical properties of the SVD have been exploited in a variety of image processing applications, including image compression [1], [2]. For example, SVD offers a natural method for distinguishing the most informative components of an image based on the relative magnitudes of the singular values. Keeping only the k largest singular vectors, along with their associated left and right singular vectors, a reduced data approximation of an image can be constructed [3].

Orthogonal Vector Interpolation (OVI) is motivated by the recognition that an operation applied to each column vector, U_i or V_i, of the SVD of an image matrix only affects image features in the projection associated with σ_i, while interpolation of the rows creates recursive compositions of the projections. More generally, the distinguishing feature of the OVI methodology is that it operates on the orthogonal vectors of a given matrix/image, not just a subset of the singular values.

For applications involving a need to alter the resolution of an image – scaling the horizontal or vertical resolution up or down – the OVI approach is to interpolate the singular vectors, i.e., the columns of U and V, from the SVD of a given $m \times n$ image matrix M to produce a $p \times q$ image P using the same matrix D of singular values, i.e., the same signature as the given image. The steps can be summarized as follows:

1) Obtain the SVD of M as $UDV^T = M$

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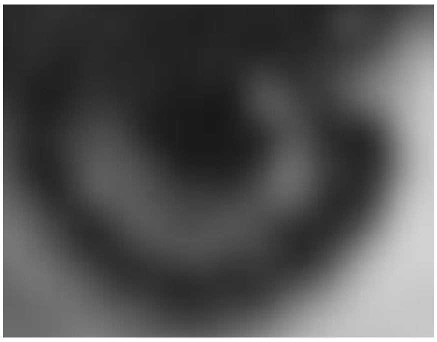
2) Construct a *p*-element interpolation of each column of U and a *q*-element interpolation of the columns of V using a chosen interpolation algorithm, e.g., cubic interpolation.
3) Let U_P and V_P be the interpolated sets of singular vectors from the previous step and construct P as P=$U_P D V_P^T$
4) Scale the transformed image to preserve a specified property of the original image, e.g., mean intensity.

The above algorithm is defined generally to apply to the conversion of an image[1] of a given resolution to one with an arbitrarily chosen resolution, e.g., a lower resolution for image decimation or higher for super-resolution. Different vector interpolation algorithms will in general produce different results. For example, linear interpolation between consecutive elements will only produce a simple linear interpolation of the image. By contrast, a high-order polynomial interpolation will tend to produce complex structures within the generated detail of the interpolated image [4], [5].

Experiments reveal that OVI produces results that are comparable to widely-used decimation and image superresolution (ISR) methods. Bicubic weighted-average interpolation is the most widely used ISR method because it produces highly-consistent results across a wide spectrum of different image types despite its characteristic blurring of detail [6]. Comparisons of bicubic and OVI on the standard 512 × 512 *Lena* image in Fig. 1 and Fig. 2 show that OVI produces visibly less blurring than bicubic.



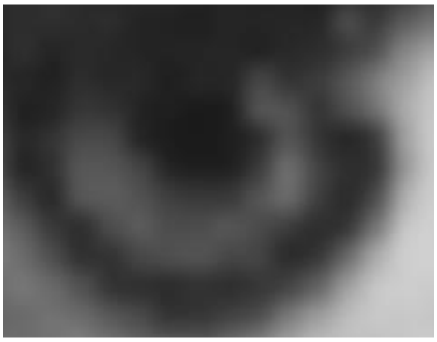


Figure 1. Top is the full Lena image. Middle is a bicubic 16x enlargement of the left-eye feature. Bottom is the 16x result obtained using the OVI method.

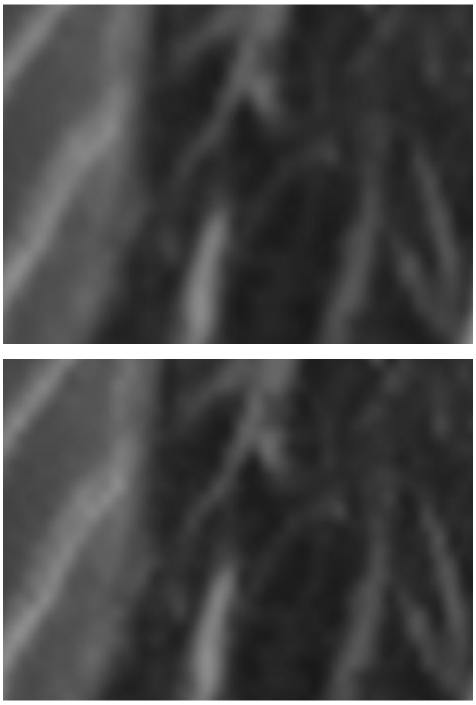


Figure 2. Top is a bicubic 20x enlargement of a region in the feathers of the hat. Bottom is the 20x OVI result for the same region. (Both of the enlarged samples have been downscaled by approximately 50% to fit the confines of the page width.)

Fig. 2 shows 20x enlargements of a small region of the feathers on the hat. Again, the OVI result seems to preserve visibly more detail/texture, i.e., less visible blurring than what is produced from bicubic interpolation.

The differences between the two methods in the examples of Fig. 1 and Fig. 2 are clearly visible when viewed on a computer screen at full size, but this level of improvement of OVI over bicubic may not be practically significant for most applications[2].

[1] The algorithm can be applied separately to each channel of a color image.

[2] Many other experiments have been performed using different vector interpolation methods; transforming interpolated vectors to the nearest set of orthogonal vectors (which also involves use of the SVD); and interpolating the rows of U and V. As is true for many of the more sophisticated ISR algorithms described in the literature, the results obtained with these variants of the basic OVI interpolation algorithm were widely varying and motivated consideration of aesthetic issues discussed in the next section.

While OVI's exploitation of global structural properties of images does not yield significant benefit for purposes of ISR, extensive experimentation in that domain has suggested a potentially promising alternative application: *aesthetically-motivated image transformations*.

III. Aesthetically-Motivated Image Transformations (AMITs)

Online mobile photo-sharing and social networking services, such as Instagram, have created a large demand for *filters* that are applied to give images various kinds specialized aesthetic qualities. A popular example is the use of a sepia-tone transformation to give an image the appearance of a photograph from the early 20th Century. Other examples replicate the qualities of halftone printing, hand-drawn line sketches, cartoons, and other "looks" that are based on existing or vintage media products.

Another category of filter design attempts to provide completely novel transformations that are not based on any pre-existing set of characteristics. These filters are often intended to attract attention specifically because the transformed results do not resemble a familiar aesthetic.

A major practical challenge to the creation of image filters is the limited variety of image properties that can be easily and efficiently manipulated using standard image processing tools. These properties include color (e.g., transforming to black-and-white or sepia-tone) and physical distortion of the image surface, e.g., to replicate the appearance of curled corners.

The OVI approach provides a completely different set of image parameters that can be altered to potentially capture a defined aesthetic with better fidelity or to create a completely new aesthetic. More specifically, the SVD of the image (1) provides parameters in the form of the rows and columns of U and V and the singular values on the diagonal of D. For example, interpolating the rows of U and V – instead of just the columns as was done for OVI interpolation in the previous section – and commensurately interpolating the singular values, i.e., so that they are dimensionally consistent (conformant) for multiplication, can produce a wide variety of effects. Computationally, SVD computation can be expensive, but there are more efficient approximations available [7], [8]. Fig. 3 provides an example of how OVI can significantly alter the structural content of an image.

Figure 3. The two images depict an image of a lion (Top) and the result of applying an OVI transformation (Bottom) which includes the interpolation of the rows of U and V.

As can be seen in the lion example, interpolation of the row vectors of U and V has the effect of replicating image structure in a way that is superficially similar to ghosting artifacts in double-exposure images. What is critical to notice, however, is that there is no obvious ghost replication of the lion's eye, nose, and mouth. What *appears* to have been generated by the transformation is additional synthetic detail consistent with the dominant feature structure in the image: the lion's fur.

A closer examination of the algorithm reveals that the interpolation of row vectors and their associated singular values does in fact have the effect of interpolating information in the transformed space of the singular values. Thus, it should not be surprising to see additional detail in the resulting transformed image. At present, though, any interpretation of the precise nature of such synthetic detail is purely speculative.

Fig. 4 shows a transformation applied to Lena that is significantly less radical than that of Fig. 3.

The transformation of Fig. 4 reveals a significant incorporation of new detail, e.g., beyond what is available from an increase in contrast. This can be seen in the mirror, where in addition to artificial texture some out-of-focus features have become de-blurred by the addition of synthetic detail. The vertical beam above the brim of the hat also appears de-blurred because of added detail. In fact, the added detail gives the appearance that the right edge of the vertical beam crosses in front of the diagonal beam, which is not the case in the original image. Moreover, the beam is also 25% *wider*. This is so striking that it is displayed more closely in Fig. 5.

Figure 4. Top is the original Lena image and Bottom is the result obtained from an OVI transformation (cropped for emphasis of detail). Synthetic texture is clearly visible in the transformed image that is not present in the original.

Figure 5. Top is a close-up of the beams in the upper-right corner of the original Lena image. Bottom is the same region after the OVI transformation. The vertical beam now appears lengthened and widened and seems to overlap the diagonal beam.

At first glance the extension in length of the vertical beam in Fig. 5 seems most surprising because it changes the apparent relative depth of the diagonal beam. However, the extension in width is arguably more remarkable because it is somehow achieved without altering the foreground boundary of the hat. A possible explanation is that these features correspond to information encoded separately in different singular values/vectors in such a way that they are transformed independently. In other words, vertically-oriented light-colored features may be more dominant in the image and are jointly enhanced, thus causing the vertical beam (and its shadow) to become merged into a single vertical feature.

Fig. 6 gives an example of an extreme OVI transformation applied to a color image.

It should be noted that OVI transformations may also prove useful as an intermediate step of other image processing algorithms. For example, the OVI-generated intensities of Lena in Fig. 4 can be merged with the softfocus color image of Lena (from Fig. 6) to produce a more “natural-looking” result as depicted in Fig. 7.

Fig. 7 provides an example of how OVI can be applied to assist with commonly-performed image processing operations. In this case OVI provides texture and detail that does not exist in the original to achieve something resembling the look of a raw original photograph. It is straightforward to verify that standard processing tools cannot be applied to the original Lena image to replicate the OVI result.

Figure 6. Top is the original Lena image in color and Bottom is an OVI transformation of the image. The result shows how texture and colors can be transformed in non-local ways. For example, the left eye is purple and the right eye is green whereas both are brown in the original image.

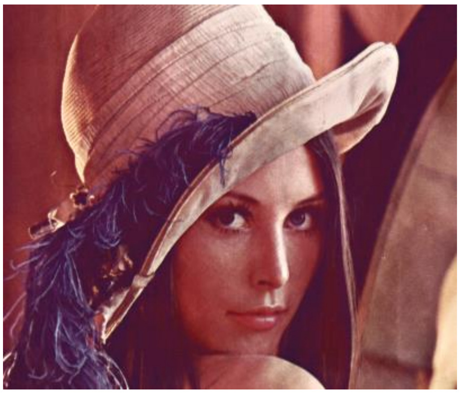


Figure 7. Top is the original Lena image in color and Bottom is an image obtained by merging it with the OVI greyscale result from Fig. 6. The result is an image with features consistent with a raw photograph prior to the application of air-brushing and other polishing.

IV. Discussion

In this paper we have defined Orthogonal Vector Interpolation (OVI) for aesthetically-motivated image transformation (AMIT). Results suggest that OVI has very significant promise as a tool for creating aesthetically interesting image transformations (e.g., filter apps) for image sharing on social media. Several examples have been presented but there is clearly need for more work to characterize how specific vector operations performed on the rows and columns of the orthogonal matrices (and the diagonal vector of singular values) qualitatively affect the resulting transformed image.

In summary, the novel feature of the OVI approach is that it treats all elements of the singular-value decomposition of an image as parameters for transforming that image. ISR and AMIT applications have been examined, but evidence (e.g., from Fig. 7) suggests potential uses in other areas of image processing and computational photography.

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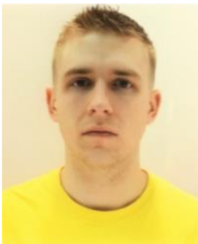

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