# Edge-Preserving Image Denoising Based on Orthogonal Wavelet Transform and Level Sets

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Abstract-The level set approach has the potential to accomplish simultaneous noise reduction and edge preservation when it is used for image denoising. However, this kind of techniques is not very efficient for denoising very noisy images for their non-reliable edge-stopping criterion in the Partial Differential Equation (PDE). In addition, the numerical calculation of curvature and other partial derivatives in the PDE is very sensitive to noise. In this paper, a new algorithm is developed to tackle such problems. Our idea is to first decompose the noisy image with the Orthogonal Wavelet Transform (OWT) and then we only filter the noisy wavelet coefficients at the three finest scales without touching the wavelet coefficients at higher levels for reducing noise while preserving edgerelated coefficients. The level-set based curve evolution is finally performed on the less-noisy image reconstructed from the denoised wavelet coefficients. Thus, the PDE model can be optimized by removing the Gaussian smoothing component. Furthermore, the numerical calculation of all partial derivatives in the PDE is influenced by less noise and the selective denoising becomes more efficient. Experimental results show that the proposed algorithm outperforms the conventional level set methods and generates state-of-the-art denoising results in edge preservation and noise reduction.

*Index Terms*—orthogonal wavelet transform, level sets, mean curvature, image denoising

# I. INTRODUCTION

The objective of image denoising is trying to recover the noise-free images from their noisy observations. However, how to preserve edges when reducing noise is a critical challenge for state-of-the-art image denoising techniques. Traditional image denoising techniques, such as linear Gaussian smoothing and low-pass filtering, can reduce noise, but edges are also blurred since edges are present in high frequencies. The wavelet-based hardthresholding techniques can eliminate much of noise by setting the small magnitude coefficients to zero, however artifact of Gibbs oscillation near discontinuities is usually introduced. Although the wavelet-based softthresholding techniques [1], [2] greatly improve the hardthresholding techniques by significantly reducing Gibbs oscillation, Gibbs oscillation cannot be eliminated. As a effectiveness of the result. the wavelet-based thresholding techniques is limited for edge-preserving image denoising applications, such as medical image denoising. In recent 10 years, the level-set based nonlinear denoising methodologies have been a very interesting research topic in image processing [3]-[7]. This class of denoising techniques is in general very efficient to preserve image edges for piecewise-smooth images separated by edges because the curvaturedependent evolution is only encouraged in the smooth regions, and it is automatically inhibited across edges. Thus, the level-set based denoising techniques can achieve simultaneous noise reduction and edgepreservation. However, they are only efficient for denoising those images that are corrupted by a low level of noise. They are not very efficient for smoothing very noisy images for the lack of a reliable edge-stopping criterion in the PDE and for the noise-sensitivity of the partial derivatives in the PDE as analyzed below. As a result, noise cannot be reduced effectively. To reduce the influence of noise on the level set methods for noise reduction, a wavelet-based multiscale level-set curve evolution is proposed [8]. The noisy image is first decomposed into a linear scale-space using the dyadic overcomplete wavelet transform [9]. Afterwards the finest scale of the scale-space is filtered by using the MMSE-based method, making the linear scale-space even more stationary. Finally, the curvature-dependent evolution is performed on the scale-space. Since for a piecewise-constant image, the scale-space is still piecewise-constant and is more stationary than the original noisy image, the wavelet-based multiscale levelset curve evolution is more efficient than the conventional level set methods. However, the computational complexity is expensive. Our motivation of using the OWT in this paper is to reduce the computational complexity while retaining its denoising efficiency.

To leverage the edge-preserving property of level sets in image denoising while circumventing its limitations of non-reliable edge-stopping criterion and noise-sensitivity, we develop a new algorithm in this paper. We propose to

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first convert the noisy image into a less-noisy one by decomposing the noisy image with the orthogonal wavelet transform and only filtering the noisy wavelet coefficients at the three finest scales. Finally, the curvature-dependent diffusion is performed on the lessnoisy image reconstructed from the denoised wavelet coefficients rather than directly on the original noisy image or on the wavelet coefficients. The benefit of using the first pass of wavelet-based denoising is that we can convert a very noisy image into a much less noisy one while preserving its edges as much as possible. This makes it possible for us to make full use of the power of level sets in selective smoothing when the PDE model is used as the second pass of denoising. Also, the PDE model can be further optimized by removing the Gaussian filtering component, and the numerical calculations of the curvature and other partial derivatives become more reliable. People may argue why not perform the curvature-dependent evolution on the orthogonal wavelet coefficients as done with the overcomplete wavelet transform [9] The point is that if we do so, it is easy to cause the Gibbs oscillation in the denoised image since the orthogonal wavelet transform is not translation-invariant, but the overcomplete wavelet transform is translation- invariant. Comparative studies have demonstrated that the proposed algorithm can significantly improve SNR while preserving edges well. The proposed algorithm outperforms the state-of-the-art level-set based nonlinear denoising techniques.

This paper is organized as follows: Section II describes the orthogonal wavelet transform and Section III describes the related work about level sets in image denoising. We present the details of the proposed algorithm in Section IV. The experimental results are demonstrated in Section V and conclusions are made in Section VI.

#### II. THE ORTHOGONAL WAVELET TRANSFORM

In this work, the Orthogonal Wavelet Transformation (OWT) is used for Multiresolution Analysis (MRA). In MRA, a function is usually viewed at various levels of approximations or at various resolutions. This makes MRA possible to decompose a complicated function into several simpler ones, each of which is more convenient for analysis. The discrete wavelet transform can be viewed as a kind of MRA and its basic idea is to project a function or a signal  $f(x) \in L^2(R)$  onto a sequence of closed successive octave approximation subspaces  $V_i$ and their associated detail subspaces  $W_i$  with multiple resolutions. At each resolution  $2^{-j}$ , the approximation subspace  $V_i$  and its associated detail subspace  $W_i$ contain the necessary information to reconstruct the approximation subspace  $V_{i-1}$  at the next finer resolution  $2^{-j+1}$  .

In constructing wavelets, under certain conditions, both scaling function and wavelet function can be implemented by the low-pass filter  $\{h_k\}$  and the highpass filter {  $g_k$  }, respectively [10]. A one-dimensional (1-D) signal  $s_0$  can be recursively decomposed into a sequence of lower resolution approximations {  $s_{j,k}$  ;  $j,k \in \mathbb{Z}$  } and details {  $d_{j,k}$  ;  $j,k \in \mathbb{Z}$  } using the following fast Discrete Wavelet Transform (DWT) [10]:

$$s_{j,k} = \sqrt{2} \sum_{n}^{n} h_{n-2k} s_{j-1,n}$$

$$d_{j,k} = \sqrt{2} \sum_{n}^{n} g_{n-2k} s_{j-1,n}$$
(1)

While the following fast inverse DWT can be used to reconstruct the original image,

$$s_{j-1,n} = \sqrt{2} \left( \sum_{k} h_{n-2k} s_{j,k} + \sum_{k} g_{n-2k} d_{j,k} \right)$$
(2)

The 1-D WT can be easily extended to twodimensional (2-D) WT for image processing. In the 2-D WT, there are three high-pass filters: 1) high-pass in *x* but low-pass in *y* direction,  $g_{HL}(k,l) = g(k)h(l)$ , 2) lowpass in *x* but high-pass in *y* direction,  $g_{LH}(k,l) = h(k)g(l)$ , and, 3) high-pass in both *x* and *y* directions,  $g_{HH}(k,l) = g(k)g(l)$ . As a result, a 2-D image can be decomposed into a pyramidal structure with lowlow (*LL*), low-high (*LH*), high-low (*HL*), and highhigh (*HH*), spatially oriented frequency channels as shown in Fig. 1. The details about how to construct the filters and the implementation of the fast WT are referred to [10], [11].

LL <sub>3</sub> HL <sub>3</sub> LH <sub>3</sub> HH <sub>3</sub>	HL <sub>2</sub>	$HL_1$
$LH_2$	$HH_2$	
LH <sub>1</sub>		HHı

Figure 1. A 3-level decomposition example about the 2-D orthogonal WT.



Figure 2. An example of 3-level 2-D orthogonal WT. (a) is the image and (b) is the orthogonal WT result.

A 3-level decomposition example about the 2-D orthogonal WT of a piecewise-constant image is shown in Fig. 2. In this work, the Daubechies' length-8 wavelet  $D_4$  [11] is used for the orthogonal WT for its proper regularity and fast processing.

# III. REVIEW OF LEVEL-SET BASED IMAGE DENOISING

In the recent 20 years, the level set methods [12]-[14] have been received great attention in image processing. The basic idea of the level set in image processing is to represent the evolving image intensity surface as a hypersurface  $\gamma(t)$ , and embed this hypersurface as the zero level set of a higher dimensional function  $\phi$ , defined by  $\phi(x, y, t) = d$ , where d is the signed distance from the point (x, y) to the hypersurface  $\gamma(t)$ . When evolving the hypersurface  $\gamma(t)$ , any point  $x(t) \in \mathbb{R}^N$ satisfies  $\phi(x(t),t) \equiv 0$ . A Eulerian formulation is produced for the motion of the surface, propagating along its normal direction at a given speed F(x), where F(x) is a function of the surface characteristics (curvature and normal direction at the point x(t)), and the image characteristics (intensity value and gradient). According to the chain rule, the evolution of  $\phi$  can be given as follows:

$$\phi_t + F |\nabla \phi| = 0 \tag{3}$$

With the initial condition  $\phi(x, t = 0) = \phi_0$ . The implicit level-set approach is more advantageous over the explicit parameterized contour models like snakes [15] to represent curves or surfaces. The speed function *F* can be decomposed into an advective speed and a diffusive speed:

$$\phi_t + g \cdot (1 - \varepsilon \cdot K) |\nabla \phi| = 0 \tag{4}$$

where  $\varepsilon$  is a small constant and K is the mean curvature:

$$K = \frac{\phi_{xx}\phi_{y}^{2} + \phi_{yy}\phi_{x}^{2} - 2\phi_{x}\phi_{y}\phi_{xy}}{(\phi_{x}^{2} + \phi_{y}^{2})^{\frac{3}{2}}}$$
(5)

Generally, the edge-stopping criterion function g is taken as [7]:

$$g(|\nabla G_{\sigma} * I_0(x)|) = e^{-\alpha |\nabla G_{\sigma} * I_0(x)|}, \alpha > 0$$
(6)

where,  $I_0(x)$  is the initial image and  $G_{\sigma}$  is the Gauss filter with the scale parameter  $\sigma$ , and  $\nabla$  denotes the gradient operator. The edge-stopping criterion function g is designed in such a way that its value is close to zero when the point x is located at edges indicated by high gradients, and its value is close to one when the point xis within a homogeneous region indicated by low gradients. With g, the evolution is only encouraged within a homogeneous region or along the edge direction, but it is inhibited across edges. Thus, edges can be preserved over time t for noise reduction. For image denoising, the level set methods have the similar idea with the anisotropic diffusion [16], but they accomplish it in a different way. Since for image denoising, it is desirable that the diffusion in the gradient direction is very small, by deleting the constant speed term in the PDE in (4), the PDE for selective image smoothing becomes [6]:

$$\phi_t = g(|\nabla I|) \cdot \varepsilon \cdot K \cdot |\nabla \phi| \tag{7}$$

To further improve the accuracy of evolution when it approaches to edges, an additional constraint is usually added into the PDE [17],

$$\phi_t = g(|\nabla I|) \cdot \varepsilon \cdot K \cdot |\nabla \phi| + \beta \cdot \nabla g \cdot \nabla \phi \tag{8}$$

where  $\beta$  is a constant. According to the curvaturedependent evolution process, the image smoothing process can be defined as following:

 $I(x, y, t+1) = I(x, y, t) + \Delta t(g(|\nabla I|) \cdot K \cdot |\nabla I| + \beta \cdot \nabla g \cdot \nabla I)$ (9) where the original noisy image is used as the initial condition I(x, y, 0), (x, y) denotes a pixel position to be smoothed in a 2-D image domain, t denotes the discrete time steps (iterations),  $\Delta t$  is a small number to control the stability of the PDE, and for  $g(|\nabla I|)$ ,  $|\nabla I|$  is calculated from the Gaussian smoothed noisy image. However, when the curvature-dependent evolution model is directly applied to a noisy image as done in all conventional level-set based denoising techniques, noise cannot be reduced efficiently. This is for the facts that in (6), the gradients for determining the edge-stopping function values are calculated from a smoothed image by using the Gaussian filtering method. However, the Gaussian filtering usually gets edges blurred when reducing noise. Thus, the obtained edge-stopping criterion is not reliable for very noisy images. In addition, the mean curvature defined in (5) contains the first- and second-order partial derivatives. When the image I(x, y, 0) is corrupted by a high level of noise, in the right-hand side of PDE (9), the mean curvature and gradient measurements in the second term  $\Delta t(g(|\nabla I|) \cdot K \cdot |\nabla I|)$  are very sensitive to noise. Also, for the third term  $\beta \cdot \nabla g \cdot \nabla I$ , due to the noise, the vectors may deviate from the actual gradient directions, resulting in inefficient edge preservation.

### IV. THE PROPOSED DENOISING ALGORITHM

The proposed algorithm in this paper includes two components: the first pass of wavelet-based denoising for the 3 finest scales, and the second pass of the level set based denoising on the reconstructed images from the denoised wavelet coefficients in the first pass.

#### A. Statistical Filtering on the Orthogonal Wavelet Transform Domain

When the noisy image is represented with the orthogonal wavelet transformation, we use the Minimum Mean Squared Error (MMSE)-based filtering method to reduce noise in the wavelet coefficients, but it is only performed on the three finest scales. The rationale is that statistical noise in the spatial image domain is a kind of random oscillation and in the orthogonal wavelet transform domain, noise is amplified to be the high frequency information and is mostly located at the fine scales. So, it is not efficient to do the curvature-dependent evolution on the 3 noisy finest scales. Suppose an image f(x, y) is corrupted by the Additive White

Gaussian Noise (AWGN) with variance  $\sigma_n^2$ . Let the observed noisy image  $f_n(x, y)$  be represented as:

$$f_n(x, y) = f(x, y) + n(x, y)$$
 (10)

For  $x, y = 0, 1, 2, \dots M - 1$ , where n(x, y) is the noise term, (x, y) is the spatial position, and M is the image dimension in the row and column directions. For the chosen orthogonal wavelet transform, the transformed noisy image  $f_n(x, y)$  can be written as:

$$w_{2^{j}}^{k} f_{n}(x, y) = w_{2^{j}}^{k} f(x, y) + w_{2^{j}}^{k} n(x, y), k = 1, 2, 3, j = 1, 2$$
(11)

where  $w_{2^{j}}^{k} f(x, y)$  denotes the orthogonal wavelet coefficient of the noise-free image at location (x, y) and scale  $2^{j}$  with the subband orientation k = 1, 2, 3, in the horizontal, vertical and diagonal direction, respectively, as shown in Fig. 1, and Fig. 2(b), respectively.  $w_{2^{j}}^{k} f_{n}(x, y)$  denotes the wavelet coefficient of the noisy image, and  $w_{2^{j}}^{k} n(x, y)$  denotes the wavelet transform coefficient of the zero-mean and  $\sigma_{n}^{2}$ -variance, Additive White Gaussian Noise (AWGN). It is still an AWGN due to the orthonormality of the OWT.

Motivated by the LAWMAP algorithm [18], in this work, the wavelet coefficients  $w_{2^{j}}^{k} f(x, y)$  at the three finest scales ( $j \le 2$ ) of the noise-free image are assumed to be the conditionally independent zero-mean Gaussian random variables  $N(0, \sigma_{(x,y)}^2)$ , given their variances  $\sigma_{(x,y)}^2$ . These variances  $\sigma_{(x,y)}^2$  are modeled as identically distributed, highly correlated random variables. According to the Maximum Likelihood (ML) estimation, the local variance  $\sigma_{(x,y)}^2$  is obtained from the local noisy wavelet coefficients as following [18]:

$$\hat{\sigma}_{(x,y)}^2 = Max\{0, \frac{1}{|\eta_{(x,y)}|} \sum_{(p,q) \in \eta_{(x,y)}} [w_{2^j}^k f_n(x,y)]^2 - \sigma_n^2\}$$
(12)

where  $\eta_{(x,y)}$  denotes the spatial neighborhood of the position of  $w_{2'}^k f_n(x,y)$ ,  $|\eta_{(x,y)}|$  denotes the number of neighbors in  $\eta_{(x,y)}$ . The neighborhood  $\eta_{(x,y)}$  is defined as a square window centered at the position of  $w_{2'}^k f_n(x,y)$ . The noise standard deviation  $\sigma_n$  is estimated separately using a robust estimation, the median absolute deviation of wavelet coefficients at the finest scale diagonal subband divided by 0.6745 [18],

$$\hat{\sigma}_n = Median(|w_2^3 f_n(x, y)|) / 0.6745$$
 (13)

After the variance of local noise-free wavelet coefficients is estimated, the noise-free wavelet coefficient value of  $w_{2^{j}}^{k} f(x, y)$  is estimated as following [18]:

$$\hat{w}_{2^{j}}^{k}f(x,y) = \frac{\hat{\sigma}_{(x,y)}^{2}}{\hat{\sigma}_{(x,y)}^{2} + \hat{\sigma}_{n}^{2}} w_{2^{j}}^{k} f_{n}(x,y)$$
(14)

#### B. Level-Set Based Curve Evolution

In this work, we still use the level set methods defined in (9). However, after the noisy image I has been converted into a less-noisy one,  $\tilde{I}$ , for determining the edge-stopping function values, we propose to calculate the gradients directly from the less-noisy image rather than from an external force field of Gauss-smoothed image. Thus, by removing the Gaussian smoothing component from the PDE in (9), the PDE can be optimized into:

$$\widetilde{I}(x, y, t+1) = \widetilde{I}(x, y, t) + \Delta t(g(|\nabla \widetilde{I}|) \cdot K \cdot |\nabla \widetilde{I}| + \beta \cdot \nabla g \cdot \nabla \widetilde{I})$$
(15)

The edge-stopping criterion g is defined as [16]:

$$g(x) = 1 / (1 + \frac{x^2}{k^2})$$
(16)

where, x is the gradient magnitude calculated directly from the less-noisy image, and k is a threshold. Furthermore, the curvature  $\kappa$  and gradient in (15) are affected by less noise and they become much more reliable. Therefore, the PDE in (15) becomes more robust than that when it is directly applied to the noisy image. The numerical implementation of (15) is as follows:

$$\widetilde{I}(x, y, t+1) = \widetilde{I}(x, y, t) + \Delta t [g_{x,y}^{t} \langle | \nabla \widetilde{I}(x, y) \rangle \cdot K_{i,j} \cdot ((\delta_{x} \widetilde{I}_{x,y}^{t})^{2} + (\delta_{y} \widetilde{I}_{x,y}^{t})^{2} \rangle^{\frac{1}{2}} + \beta \{ \max((g_{x})_{x,y}^{t}, 0) \delta_{x}^{-} \widetilde{I}_{x,y}^{t} + \min((g_{x})_{x,y}^{t}, 0) \delta_{x}^{+} \widetilde{I}_{x,y}^{t} + \max((g_{y})_{x,y}^{t}, 0) \delta_{y}^{-} \widetilde{I}_{x,y}^{t} + \min((g_{y})_{x,y}^{t}, 0) \delta_{y}^{+} I_{x,y}^{t} \}$$
(17)

where

$$\delta_x^+ \widetilde{I}_{x,y}^t = \widetilde{I}_{x+1,y}^t - \widetilde{I}_{x,y}^t, \ \delta_x^- \widetilde{I}_{x,y}^t = \widetilde{I}_{x,y}^t - \widetilde{I}_{x-1,y}^t$$
(18)

$$\delta_{y}^{+}\widetilde{I}_{x,y}^{t} = \widetilde{I}_{x,y+1}^{t} - \widetilde{I}_{x,y}^{t}, \ \delta_{y}^{-}\widetilde{I}_{x,y}^{t} = \widetilde{I}_{x,y}^{t} - \widetilde{I}_{x,y-1}^{t}$$
(19)

$$\delta_x \widetilde{I}_{x,y}^t = (\widetilde{I}_{x+1,y}^t - \widetilde{I}_{x-1,y}^t)/2, \ \delta_y \widetilde{I}_{x,y}^t = (\widetilde{I}_{x,y+1}^t - \widetilde{I}_{x,y-1}^t)/2$$
(20)

C. Summary of the Proposed Algorithm

- The proposed algorithm can be summarized as follows:
- 1. Decompose the noisy image into three levels using the orthogonal WT.
- 2. For the orthogonal WT coefficients at the three finest scales, do noise reduction using the adaptive statistical analysis method described in Section 4.1. We have tried to apply the curve evolution to the wavelet coefficients at all levels, but it is found less efficient than the way done in this paper.
- 3. Reconstruct the denoised image with the inverse orthogonal WT.
- 4. Apply the level-set based curve evolution model described in Section 4.2 to the less-noisy image obtained in Step 3.

The proposed algorithm is called WT\_LSCE and that without containing step 4 is called WT\_MMSE.

# V. EXPERIMENTAL RESULTS

The performance of the proposed algorithm is evaluated using the 512x512 testing images of Peppers, Lena, and Barbara. The additive white Gaussian noise with different noise variances is added to these images. The Peak Signal to Noise Ratio (PSNR) is used to evaluate the quality of the denoised images:

$$PSNR = 10 * \log_{10} \frac{255^2}{\sum_{i,i} (\hat{I}_{i,j} - I_{i,j})^2}, 0 \le i, j \le 511$$
(21)

In which  $\hat{I}$  represents the final denoised image and Irepresents the original noise-free image. For demonstrating the effectiveness of the proposed WT\_LSCE algorithm in noise reduction and edge preservation, it is compared with the counterparts of the WMLSCE MMSE [8], the Level-Set based Curve Evolution method (LSCE) [6], and the LAWMAP [18]. The PSNR values of the three noisy images with respect to different noise variances are listed in Table I. Since for a lot of state-of-the-art level-set based denoising techniques, their denoised images and numerical results for these images are not available for a complete comparison, they are not compared here.

TABLE I. THE PSNR (IN DB) OF THE NOISY STANDARD TESTING IMAGES OF LENA, BARBARA AND PEPPERS WITH RESPECT TO DIFFERENT NOISE VARIANCES

Image	PSNR (dB) vs. Noise variance ( $\sigma^2$ )			
	225	400	625	
Lena	24.66	22.18	20.27	
Barbara	24.67	22.19	20.29	
Peppers	24.81	22.36	20.47	

The PSNR values of the denoised images for the 4 algorithms with respect to different noise variances are listed in Table II, from which we can see that the proposed WT\_LSCE algorithm achieves much better denoising performance than all other algorithms. For visual quality comparison, the denoised images of the WT\_LSCE, WT\_MMSE, the LSCE [6], and the WMLSCE\_MMSE [8] algorithms, corresponding to noise variance 225 are displayed in Fig. 3 to Fig. 6, respectively, for the image Lena. In addition, the WT LSCE is much faster than both WMLSCE MMSE and LSCE. This is for the facts that in the WT LSCE algorithm, the first step of denoising in the orthogonal WT domain is very fast and when the curvaturedependent evolution is performed on the less-noisy image, a smaller number of iterations is needed for the level-set based curve evolution than that for the LSCE algorithm [6]. Since the LSCE is directly applied to the noisy image with the limitations analyzed above, its performance is the lowest in all these algorithms. For the WMLSE\_MMSE, since the curvature-dependent evolution is performed in the overcomplete wavelet transform domain, it is natural that it is slower than the proposed WT\_LSCE algorithm.

For illustrating the impact of the MMSE-based filtering on the performance of the proposed WT\_LSCE algorithm, the WT\_MMSE is tested. Its PSNR values for the 3 denoised images are listed in Table II. The WT\_MMSE scheme is more efficient than the

LAWMAP [18] algorithm, in which the MMSE-based filtering is applied to all five levels. This conforms to our analysis that the zero-mean Gaussian distribution assumption is not adequate for the orthogonal wavelet coefficients at coarser scales. So, the MMSE-based filtering is very helpful for the proposed WT\_LSCE algorithm to outperform the LSCE and LAWMAP algorithms. From the experiments, we also can see that the LSCE is not very efficient for denoising the image of Barbara. It is for the fact that the image contains a lot of textures, which are not piecewise-constant and not very suitable for the level set methods for noise reduction. However, with the proposed WT\_LSCE algorithm, we can still achieve very satisfactory denoising performance due to the MMSE-based filtering at the three finest scales.

TABLE II. PERFORMANCE (PSNR IN DB) OF THE PROPOSED WT\_LSCE COMPARED WITH THAT OF THE LSCE [6], LAWMAP [18], WMLSCE\_MMSE [8], AND WT\_MMSE ALGORITHMS FOR DIFFERENT IMAGES WITH RESPECT TO DIFFERENT NOISE VARIANCES. THE RESULTS OF LSCE ARE FROM THE AUTHOR'S IMPLEMENTATION RATHER THAN FROM THE ORIGINAL PAPER [6]

Scheme	Image	PSNR (dB) vs. Noise variance $(\sigma^2)$		
		225	400	625
WT_LSCE	Lena	32.88	31.60	30.60
	Barbara	30.56	28.97	27.76
	Peppers	32.38	31.16	30.16
LSCE	Lena	31.36	30.01	28.98
	Barbara	27.61	26.09	24.92
	Peppers	31.12	29.92	29.00
LAWMAP	Lena	32.27	30.92	29.90
	Barbara	30.13	28.57	27.40
WMLSCE_ MMSE	Lena	32.64	31.33	30.32
	Barbara	29.76	28.09	26.83
	Peppers	32.35	31.01	29.94
WT_MMSE	Lena	32.44	30.93	29.77
	Barbara	30.52	28.91	27.68
	Peppers	31.78	30.29	29.04



Figure 3. The denoised image of Lena using the proposed WT\_LSCE algorithm.



Figure 4. The denoised image of Lena using the WT\_MMSE algorithm in this paper.



Figure 5. The denoised image of Lena using the LSCE algorithm [6].



Figure 6. The denoised image of Lena using the WMLSCE\_MMSE algorithm [8].

#### VI. CONCLUSION

We have presented a very efficient algorithm to improve the level set methods for noise reduction. We first convert the noisy image into a less-noisy one with edges preserved by decomposing the noisy image using the orthogonal wavelet transform and denoising the wavelet coefficients at the three finest scales using the MMSE-based filtering. Thus, an environment is constructed so that the PDE model is influenced by much less noise and is much more robust for noise reduction and edge preservation. In addition, the PDE can be optimized by removing the Gaussian smoothing component. Experimental results show that this algorithm can achieve both very high PSNR values and very satisfactory visual quality for the denoised images. This algorithm is very efficient for medical CT & MR imaging denoising where the critical requirement is to preserve the image edges in the denoised image.

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