Algorithm of 3D Spatial Coordinates Measurement Using a Camera Image

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Abstract—**In many researches for image processing technology, 3D coordinates measurement can be done by using laser range finders or stereo vision sensors. However, it has a disadvantage of high cost and large and heavy sensors. This study proposes an algorithm of transformation of 2D positions in a camera image to 3D spatial coordinate values. The coordinates of 3 points have to be measured in advance, in order to obtain the coordinate values of unknown points. The proposed algorithm was verified through experiment. This algorithm can be used in modern industry measurement systems and development of weapon systems.**

Index Terms—**3D measurement, coordinate, monocular camera**

I. INTRODUCTION

There is widespread interest in obtaining 3D data by means of machines for the purpose of studies and applications of machine perception, pattern recognition, computer vision, robotic vision, computer aided design, apparel design, and so on. Three-dimensional data would serve to solve a wide variety of problems in such fields easily. as technology of image processing develops with computer operation speed, many researches have been done on measurement using cameras instead of human eyes. Technology of 3D measurement, image processing and recognition has been developed in industry for precision manufacturing. Among various methods, optical 3D measurement techniques have been used in many fields such as reverse engineering, quality inspection, etc. [1] A typical optical coordinate measurement is local mapping using laser range finders, which has been investigated actively.

Recently, sensors such as laser range finder, camera sensor, ultrasonic sensor and Kinect are used to perform local mapping. Although a laser range finder has high accuracy and reliability, it is not appropriate to be used in real industry, because it is very expensive. [2] Also, there is a stereo vision used widely for 3D measurement. This realizes the function of recognition of 3D space through human eyes using 2 cameras. The principle of this technology is that 3D measurement is done using 2 or more images by adjusting points in the images. [3], [4]

The monocular camera algorithm method gives 3D measurement data via a mush simpler computational scheme, so it may be more promising at this state of the art. The conventional monocular camera methods, however rely on the structured light such as CMM or SLS. Hence the SLS consists of a projector together with a camera. The projector projects a coded stripe pattern on the scene and the camera captures an image. Hence for each visible point in the world there is a corresponding stripe number (stripe value) and image location (pixel coordinates). Given the parameters of the SLS each stripe value defines a plane in the world and each pixel defines a ray in the world. The intersection of this ray and plane defines a unique 3D location in the world. The parameters of the SLS are obtained during system calibration. This is achieved by presenting a calibration reference in the field of view which features a set of fiducial marks whose spatial location is known to high accuracy. The SLS is operated normally and the pixel coordinates and stripe value for each of the fiducial marks are obtained. These triples of reference, pixel and stripe coordinates are used to estimate the unknown parameters of a mathematical model of the SLS. In this way calibration can be regarded as a parameter estimation problem. [5]-[10]

As mentioned previously, existing technologies of 3D measurement require expensive devices and complicated processes. However, transformation of 2D images or videos to 3D coordinates does not need those requirements. It can be used in 3D measurement of MAV (Micro Air Vehicle), and object trajectory measurement in weapon tests.

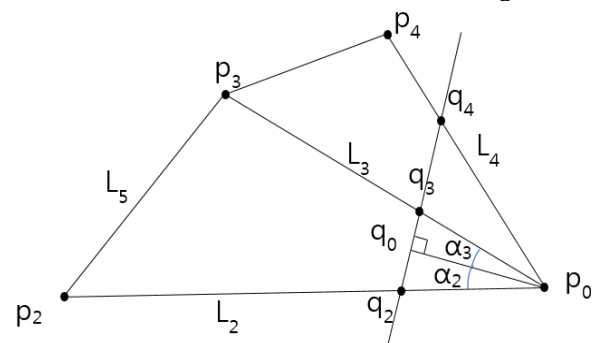In this study, an algorithm is proposed that 3D coordinates are obtained from a 2D camera image.

Figure 1. Lengths and angles of points

II. HELPFUL HINTS

To assume that the coordinates of the origin (current camera position p_0 in Fig. 1) and 3 points (p_2, p_3, p_4) are known. The coordinates of a point are obtained from the relation between the coordinates of 2D images and 3D points. The unit vectors from p0 to p_2 and from p_0 to p_3 in Fig. 1 are

$$e_2 = \frac{\overrightarrow{p_2 p_0}}{|p_2 p_0|}, \hat{e}_3 = \frac{\overrightarrow{p_3 p_0}}{|p_3 p_0|} \quad (1)$$
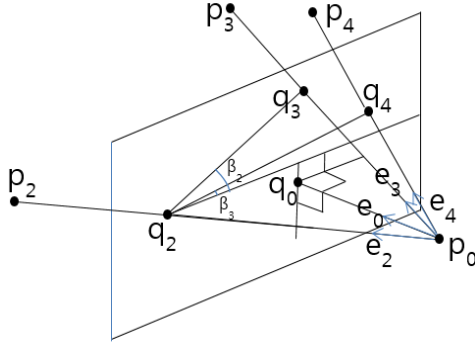

Figure 2. Positions on the screen and unit vectors

The unit vectors, e_2, e_3, are deviated from a vertical axis by $\alpha_2 = \tan^{-1}(r_2/a_0), \alpha_3 = \tan^{-1}(r_3/a_0)$ and as shown in Fig. 2, respectively. a_0 is the pixel focal length. From the two unit vectors, a unit vector corresponding to $r=0$ can be determined.

$$e_{23} = \frac{\sin \alpha_2 \hat{e}_3 - \sin \alpha_3 e_2}{\sin \alpha_2 - \sin \alpha_3} \quad (2)$$

Similarly, unit vectors can be obtained.

$$e_4 = \frac{\overrightarrow{p_4 p_0}}{|p_4 p_0|}$$

$$e_{24} = \frac{\sin \alpha_2 e_4 - \sin \alpha_4 e_2}{\sin \alpha_2 - \sin \alpha_4} \quad (3)$$

e_{23}, e_{24} are deviated from a horizontal axis by β_2, β_3 respectively. β_2, β_3 can be obtained as

$$s_{23} = \frac{\sin \alpha_2 s_3 - \sin \alpha_3 s_2}{\sin \alpha_2 - \sin \alpha_3}$$

$$s_{34} = \frac{\sin \alpha_3 s_4 - \sin \alpha_4 s_3}{\sin \alpha_3 - \sin \alpha_4} \quad (4)$$

$$\beta_2 = \tan^{-1}(s_{23}/a_0)$$

$$\beta_3 = \tan^{-1}(s_{34}/a_0)$$

From β_2, β_3, a unit vector connecting p_0 and q_0 can be obtained. This unit vector is normal to the camera picture plane.

$$e_0 = \frac{\sin \beta_2 e_{34} - \sin \beta_3 e_{23}}{\sin \beta_2 - \sin \beta_3} \quad (5)$$

An equation of a plane is expressed as $ax+by+cz=0$, and coefficients a, b, c, d can be represented by a normal vector and a point in the plane

$$e_{0x}x + e_{0y}y + e_{0z}z - (e_{0x}q_{0x} + e_{0y}q_{0x} + e_{0z}q_{0x}) = 0 \quad (6)$$

$$q_0 = p_0 + ma_0 e_0 = (p_{0x}, p_{0y}, p_{0z}) + ma_0(e_{0x}, e_{0y}, e_{0z}) \quad (7)$$

Point q_0 is represented by equation (7). m is a magnification factor which transforms a pixel value to a spatial length. The line connecting p_0 and q_i meets the plane at point q_i as shown in Fig. 2. q_2 is located in the direction of e_2 from p_0, and can be expressed as using a parameter t.

$$q_2 = p_0 + te_2 = (p_{0x}, p_{0y}, p_{0z}) + t(e_{2x}, e_{2y}, e_{2z}) \quad (8)$$

This point is on the plane, $ax+by+cz+d=0$

$$e_{0x}(p_{0x} + te_{2x}) + e_{0y}(p_{0y} + te_{2y}) + e_{0z}(p_{0z} + te_{2z})$$
$$-[e_{0x}(p_{0x} + ma_0 e_{0x}) + e_{0y}(p_{0y} + ma_0 e_{0y}) + e_{0z}(p_{0z} + ma_0 e_{0z})] = 0 \quad (9)$$

From equation (9), the parameter t can be determined as

$$t = \frac{ma_0(e_{0x}^2 + e_{0y}^2 + e_{0z}^2)}{e_{0x}e_{2x} + e_{0y}e_{2y} + e_{0z}e_{2z}} \quad (10)$$

q_i can be determined by substituting t into equation (8). There exists a transformation relation between the picture plane and the real plane. A transformation array J is

$$rs = Jq$$
$$q = J^{-1}rs \quad (11)$$

For instance, the following equation holds for $i = 2, 3, 4$.

$$\begin{bmatrix} r_i \\ s_i \\ c \end{bmatrix} = \begin{bmatrix} J_{11} & J_{21} & J_{31} \\ J_{12} & J_{22} & J_{32} \\ J_{13} & J_{23} & J_{33} \end{bmatrix} \begin{bmatrix} q_{ix} \\ q_{iy} \\ q_{iz} \end{bmatrix} (i = 2,3,4) \quad (12)$$

Or

$$r_2 = J_{11}q_{2x} + J_{21}q_{2y} + J_{31}q_{2z}$$
$$s_2 = J_{12}q_{2x} + J_{22}q_{2y} + J_{32}q_{2z}$$
$$c = J_{13}q_{2x} + J_{23}q_{2y} + J_{33}q_{2z}$$
$$r_3 = J_{11}q_{3x} + J_{21}q_{3y} + J_{31}q_{3z}$$
$$s_3 = J_{12}q_{3x} + J_{22}q_{3y} + J_{32}q_{3z} \quad (13)$$
$$c = J_{13}q_{3x} + J_{23}q_{3y} + J_{33}q_{3z}$$
$$r_4 = J_{11}q_{4x} + J_{21}q_{4y} + J_{31}q_{4z}$$
$$s_4 = J_{12}q_{4x} + J_{22}q_{4y} + J_{32}q_{4z}$$
$$c = J_{13}q_{4x} + J_{23}q_{4y} + J_{33}q_{4z}$$

$$\begin{bmatrix} q_{2x} & q_{2y} & q_{2z} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{2x} & q_{2y} & q_{2z} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{2x} & q_{2y} & q_{2z} \\ q_{3x} & q_{3y} & q_{3z} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{3x} & q_{3y} & q_{3z} \\ & \vdots & & & \vdots & & & \vdots & \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{4x} & q_{4y} & q_{4z} \end{bmatrix} \begin{bmatrix} J_{11} \\ J_{21} \\ J_{31} \\ J_{12} \\ J_{22} \\ \vdots \\ J_{33} \end{bmatrix} = \begin{bmatrix} r_2 \\ s_2 \\ c \\ r_3 \\ s_3 \\ \vdots \\ c \end{bmatrix} \quad (14)$$

r, s, q values are obtained from the pixel values. From equation (14), J can be determined. q can be determined by substituting r, s into equation (11). For example, putting the pixel coordinates r_5, s_5 into equation (3) corresponding to the point q_5 in Fig. 3, q_5 can be determined.

$$\begin{bmatrix} q_{5x} \\ q_{5y} \\ q_{5z} \end{bmatrix} = J^{-1} \begin{bmatrix} r_5 \\ s_5 \\ c \end{bmatrix} \quad (15)$$

p_5 can be obtained by determining a unit vector e_5 connecting p_0 and q_5

$$p_5 = p_0 + te_5 = (p_{0x}, p_{0y}, p_{0z}) + t(e_{5x}, e_{5y}, e_{5z}) \quad (16)$$

When p_5 is located on plane z=0, then t is obtained as

$$p_{5z} = p_{0z} + te_{5z} = 0$$

$$t = \frac{-p_{0z}}{e_{5z}} \quad (17)$$

III. EXPERIMENT AND RESULT

From 2D pixel coordinates, 3D coordinates are to be obtained using a MATLAB code based on linear algebra. The experiment was done as shown in Fig. 3.
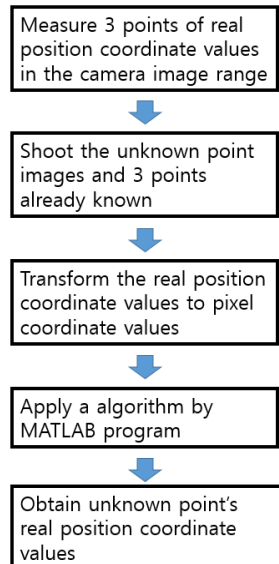
Measure 3 points of real position coordinate values in the camera image range

Shoot the unknown point images and 3 points already known

Transform the real position coordinate values to pixel coordinate values

Apply a algorithm by MATLAB program

Obtain unknown point's real position coordinate values

Figure 3. Flow chart of algorithm

TABLE I. REAL COORDINATE VALUES (UNIT: MM)

Point	x	y	z
p_2	-431.3	3194.6	1240
p_3	30	3626	630
p_4	0	3626	1240
p_5	-315	3180	820

TABLE II. PIXEL COORDINATE VALUES (UNIT: MM)

Point	x	y	z
p_2	-320	486	0
p_3	21	64	0
p_4	4	475	0
p_5	-236	163	0

For verification of the proposed algorithm, experiment was done with 4 points selected as shown in Fig. 4. The four points were on the same plane. The coordinates of the 4 points were measured in 3D real space. The results are shown in Table I. The camera position was measured from a plane z=0. A camera picture was taken with all the points in a frame. From the camera image, the pixel coordinates were measured as shown in Table II.

The pixel values and coordinated of all the points were input to the equations, and the 3D coordinates of was obtained. The distance of the points from the camera was 4m. The results are shown in Table III. The error of the measured coordinates is about 1.7%, compared with the real coordinates. This shows that the proposed algorithm is good enough.
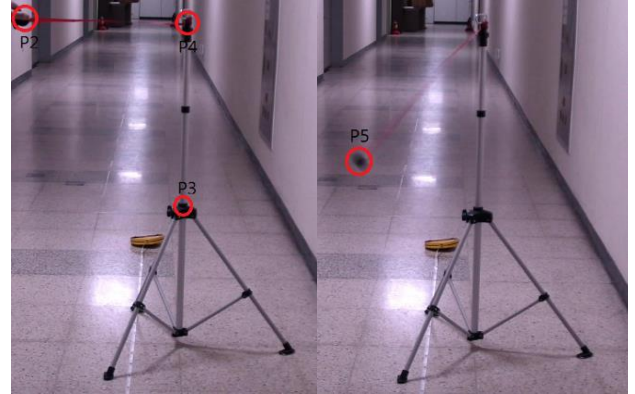
Figure 4. 2D image of test

TABLE III. RESULT OF EXPERIMENT (UNIT: MM)

Point	x	y	z	Error
Real p_5	-315	3180	820	1.7%
Computed p_5	-314	3085	837.5	

IV. CONCLUSIONS

In this study, an algorithm of determining 3D coordinates from 2D image was proposed. Experimental results showed that the measurement has about 1.7% error. This means that the algorithm is good enough. Using this algorithm, economical and fast measurement is possible.

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Hyeon Min Lee completed his B.S from Seoul National University of Science and Technology, Korea in Department of Mechanical engineering, in 2013. He is currently pursuing the MS degree in the Precision Technology Lab. of the Graduate School of Mechanical Engineering at Korea University, Korea. His study includes measurement of 3D spatial coordinates by 2D image, image processing.